First Edition

POST-TENSIONED BUILDINGS
Design and Construction

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International Edition

CHAPTER 5

10 STEPS OF DESIGN OF A POST-TENSIONED FLOOR

Olaya Post-Tensioned Towers (KSA P100)

The information and know-how necessary for design of a post-tensioned floor system were cited and discussed in detail in Chapter 4. This information is applied to two practical design examples in Chapters 6 and 7. The objective of this Chapter is to simply list the 10 steps that we will follow in design of each of the two examples. The 10 steps serve as a quick reference guide to walk us through the design.

It is recognized that, to day, most design firms use software for their routine designs, where many of the design steps listed are handled in the background by the routines of the computer program. The list and their details in the Chapters that follow also serve to better understand and evaluate the outcome of automated designs

10 STEPS OF DESIGN

1. Selection of Geometry, Sizing of the Members and Structural System
   - 1.1. Optimum span
   - 1.2. Optimum thickness, slab, column drops, slab bands, beams
   - 1.3. Selection of load path, support lines and design strips
2. Selection of Material
   - 2.1. Concrete
   - 2.2. Non-stressed reinforcement—rebar
   - 2.3. Selection of prestressing system and its design parameters
3. Selection of Loads
   - 3.1. Selfweight

3.2. Superimposed dead load
3.3. Live load, including lateral loads, if any
3.4. Temperature
4. Design Parameters
4.1. Design code
4.2. Selection of allowable stresses, exposure to corrosion and crack width, where applicable
4.3. Deflection criteria (immediate/long-term)
4.4. Fire resistivity to determine cover to reinforcement
4.5. Vibration
5. Selection of Post-Tensioning Design Parameters
5.1 Selection of Tendon Profile and Other Parameters, Such as Precompression and Percentage of load to balance
6. Selection of Analysis Procedure and Analysis, Structural Modeling, Analysis and Design
6.1. Strip method (simple frame or equivalent frame), optimization capability
6.2. Finite Elements (ADAPT-Floor Pro)
7. Extraction of Design Values and Code Check for Serviceability Limit State (SLS)
7.1. Load combinations
7.2. Stress check
7.3. Crack width control, where applicable
7.4. Minimum reinforcement
7.5. Deflection check
8. Code Check for Strength; Ultimate Limit State (ULS)
8.1. Load combination
8.2. Determination of hyperstatic actions
8.3. Calculation of design moments
8.4. Strength design for bending and ductility
8.5. Punching shear check or one-way shear
8.6. Design for transfer of column moment to slab
9. Check for Initial Condition
9.1. Load combination and determination of design values
9.2. Stress check and provision of reinforcement, where necessary
10. Structural Detailing
10.1 Detailing of rebar (rebar not determined by calculation)
10.2 Detailing of tendons

The above steps are followed by:
❖ Generation of structural drawings
❖ Generation of fabrication (shop) drawings

SPECIAL CONDITIONS
Depending on the application, or the function of the floor system special considerations, such as those listed below may be necessary

1—Vibration evaluation
2—Diaphragm design of the floor system for lateral loads
3—Participation of floor to act in frame action with walls and columns in resisting wind and seismic forces

Refer to Chapters 6 for the application of the steps listed to design of a column-supported floor system, and Chapter 7 for design of a beam frame.

CHAPTER 6

POST-TENSIONED FLOOR DESIGN STEP-BY-STEP CALCULATION

Post-Tensioned Concrete Frame under Construction
(California, P634)

FOREWORD

This example walks you through the 10 steps of design of a post-tensioned floor level of a multistory building. Each of the 10 steps is commented in detail to provide you with the background information necessary to follow the calculations.

Many aspects of the example selected, such as the arrangement of its floor supports are highly irregular. The objective in selecting an irregular structure is to expose you to the different design scenarios that you may encounter in real life structures, but you do not find covered in standard textbooks. Design conditions that are not directly encountered in this example, but are important to know, are introduced and discussed as comments.

The floor slab is provided with both column drops for punching shear, and drop panels for additional strength in resisting high negative moments over the supports. The design example also features different number of strands along the length of the structure and change in tendon profile from span to span.

Design operations that are considered common knowledge, such as the calculation of moments and shears, once the geometry of a structure, its material properties and loading are known, are not covered in detail. You are referred to your in-house frame programs for their evaluation, or other sections of the book, where the specific operations are addressed in greater detail.

The design example covers side by side both the unbonded and bonded (grouted) post-tensioning systems, thus providing a direct comparison between the design processes of the two options. In addition, in parallel, the design uses the current American building codes (ACI-318 and IBC) along with the European Code (EC2). Where applicable, reference is made to the UK's committee report TR43.

There are three methods commonly used for the design of a post-tensioned floor system—Simple Frame Methods (SFM),[1] Equivalent Frame Method (EFM); and Finite Element Method (FEM). Among the three, the EFM has been the primary method of design used by leading consulting firms over the years. However, due to its complexity, it does not lend itself to hand calculation of real structures in the environment of a consulting firm. Computer programs based on the EFM, such as ADAPT-PT are generally used. Recently, many consultants sacrifice the efficiency and the option of optimization that is feasible for designs based on EFM and opt for the benefits of FEM-based designs, such as the computer program Floor-Pro by ADAPT. These FEM-based designs can model the entire floor system and provide seamless integration of design process from architectural drawings to fabrication documents.

Hand calculations, such as the one presented herein, use the SFM.

Two text fonts are used in the following. The numerical work that forms part of the actual calculations uses the font shown below:

This font is used for the numerical work that is part of the design.

The following text font is used, wherever comments are made to supplement the calculations:

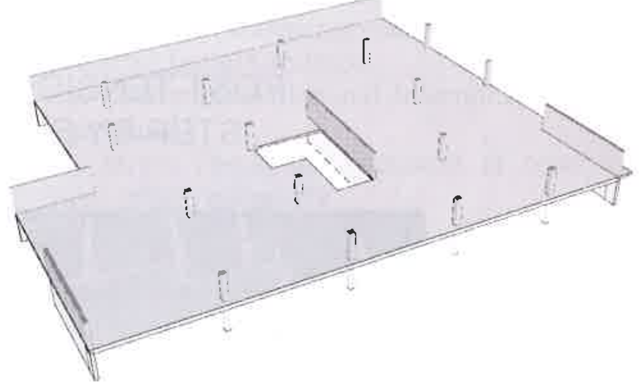
This font is used to add clarification to the calculations.

DESIGN STEPS

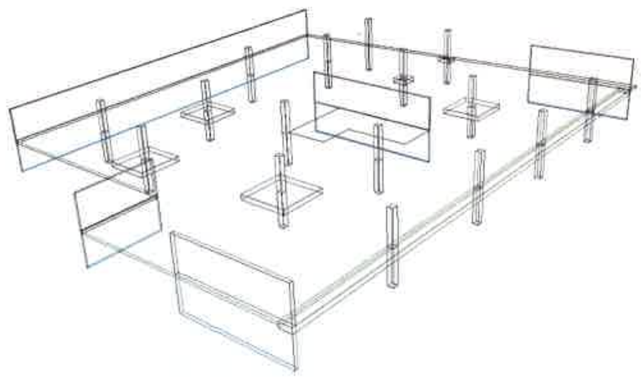
1. GEOMETRY AND STRUCTURAL SYSTEM
   1.1 Overview
   1.2 Geometry and Support Conditions
   1.3 Support Lines and Tributaries
   1.4 Idealized Design Strip
2. MATERIAL PROPERTIES
   2.1 Concrete
   2.2 Nonprestressed Reinforcement
   2.3 Prestressing

[1] The Simple Frame Method (SFM) in UK and the literature based on UK practice is referred to as "Equivalent Frame Method." It is based strictly on the cross-sectional geometry of the slab frame being designed. The term Equivalent Frame Method in the US literature is based on an approximation that is intended to simulate the two-way action of a floor slab. It is described in various versions of ACI 318.

TN7-369



(a) 3D Solid View of the Floor System (P471)



(b) See through View of the Floor System (P472)

O-1 3D Views of the Floor System

3. LOADS
   3.1 Selfweight
   3.2 Superimposed Dead Load
   3.3 Live Load
4. DESIGN PARAMETERS
   4.1 Applicable Code
   4.2 Cover to Rebar and Prestressing Strands
   4.3 Allowable Stresses
   4.4 Crack Width Limitation
   4.5 Allowable Deflection
5. ACTIONS DUE TO DEAD AND LIVE LOADS
6. POST-TENSIONING
   6.1 Selection of Design Parameters
   6.2 Selection of Post-tensioning Tendon Force and Profile
   6.3 Selection of Number of Strands
   6.4 Calculation of Balanced Load
   6.5 Determination of Actions Due to Balanced (post-tensioning) Loads
7. CODE CHECK FOR SERVICEABILITY
   7.1 Load Combinations
   7.2 Stress Check
   7.3 Crack Width Control

   7.4 Minimum Reinforcement
   7.5 Deflection Check
8. CODE CHECK FOR STRENGTH
   8.1 Load Combinations
   8.2 Determination of Hyperstatic Actions
   8.3 Calculation of Design Moments
   8.4 Strength Design for Bending and Ductility
   8.5 Punching Shear Check and Design
9. CODE CHECK FOR INITIAL CONDITION
   9.1 Load Combinations
   9.2 Stress Check
10. DETAILING

1 - GEOMETRY AND STRUCTURAL SYSTEM

1.1 Overview

Nahid building is a multi-story structure supported on walls and columns. The lateral loads are resisted by shear walls in two directions. The floor of the building is a two-way post-tensioned slab resting on columns and walls. The calculations that follow represent the design of one region of the floor slab identified by gridline B, and referred to as "design strip B." The remainder of the floor slab can be designed in a similar manner. The design is performed using the current versions of IBC; ACI-318; EC2 and TR-43.

1.2 Geometry and Support Conditions

Dimensions and Support Conditions

Floor slab dimensions are shown in Fig. 1.1-1.

❖ Slab thickness and locations of Column drops/Panels are shown in Fig. 1.1-2;

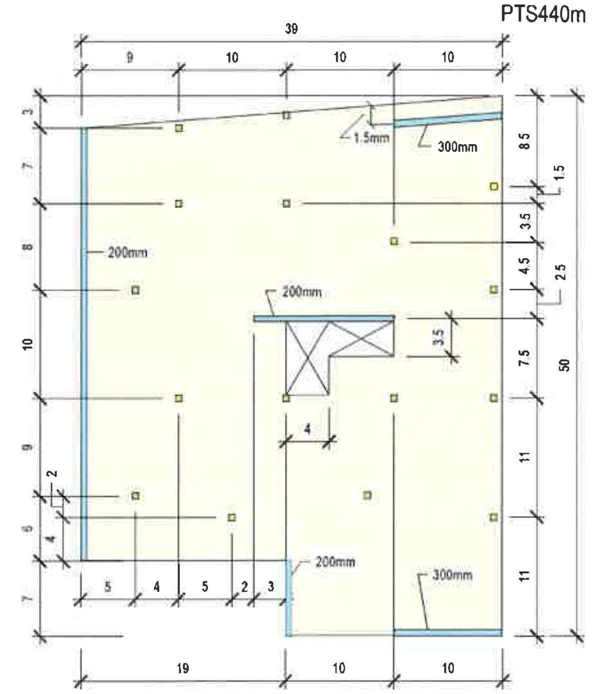
❖ Dimensions of column drops/panels shown in Fig. 1.1-3;

❖ Columns are 600 mm x 600 mm and extend above and below the slab; and

❖ Columns are assumed fixed at connection to the slab and at their far ends.

The maximum span to depth ratio for the 240 mm slab selected is less than 45, which is the upper value commonly used for similar structures. A preliminary analysis, not included in this work, showed that the slab thickness selected was not adequate for punching shear at selected column locations (marked as locations A through E in Fig. 1.2-2, and along the column supported right edge of the slab). As a result, the right edge is provided with a down turned edge beam (section ii in Fig. 1.2-2). The remainder of the locations are provided each with a column drop to resist punching shear. Further calculation of the preliminary design concluded that the required reinforcement over four of the interior columns was

PTS440m

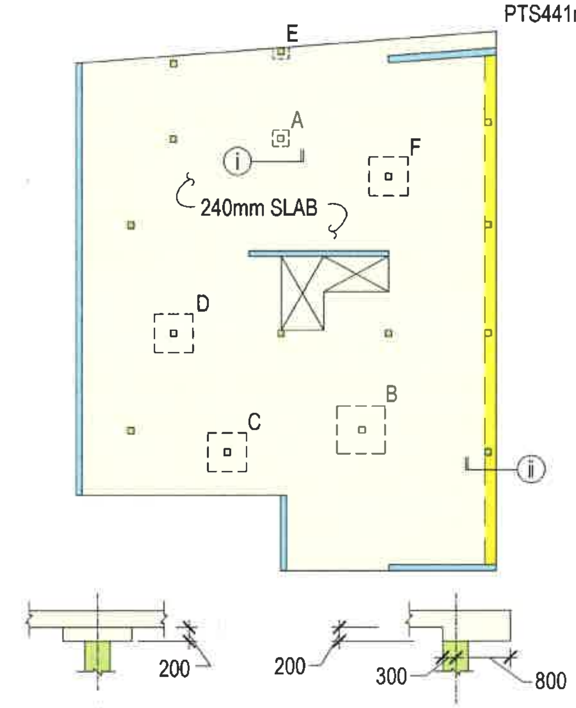


dimensions in m, uno

Floor Slab Dimensions (m)

FIGURE 1.1-1

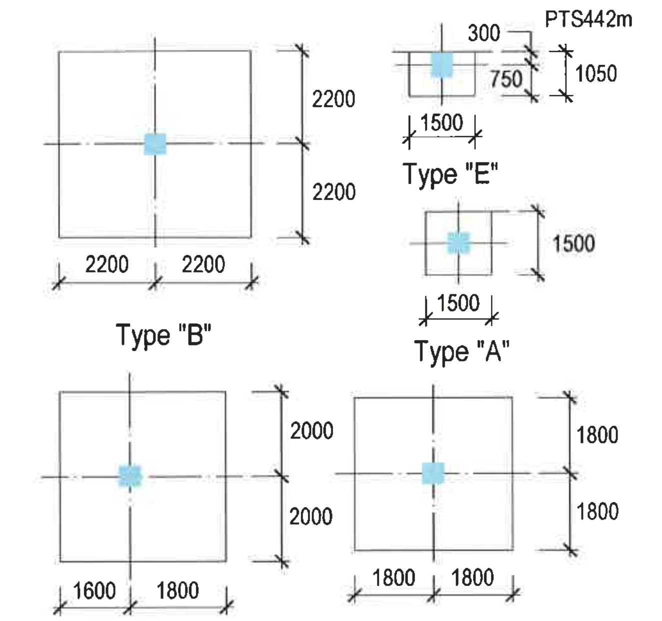
PTS441m



(i) Section - Typical (ii) Section

Thickness Geometry of the Floor (mm)

FIGURE 1.1-2



All 200mm below soffit of slab

Plan Geometry of Drop Caps and Drop Panels (mm)

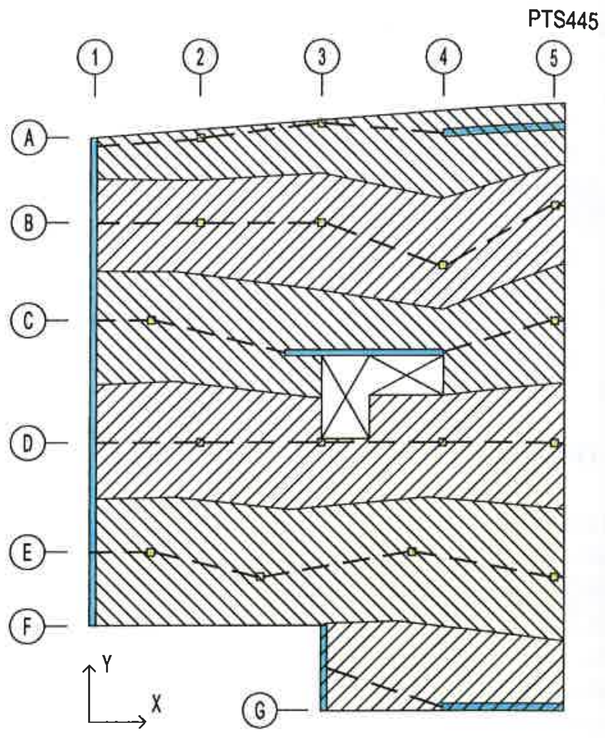
FIGURE 1.1-3

excessive (more than 4200 mm2) [Aalami, 1989]. To avoid congestion of top reinforcement, the column drops at these locations were enlarged to qualify them as drop panels. These locations are marked as B, C, D and F in Fig.1.2-2. While it is practical to eliminate column drops at locations A and E through provision of punching shear reinforcement, the drop panels cannot be eliminated without causing congestion in top rebar.

1.3 Support Lines and Tributaries

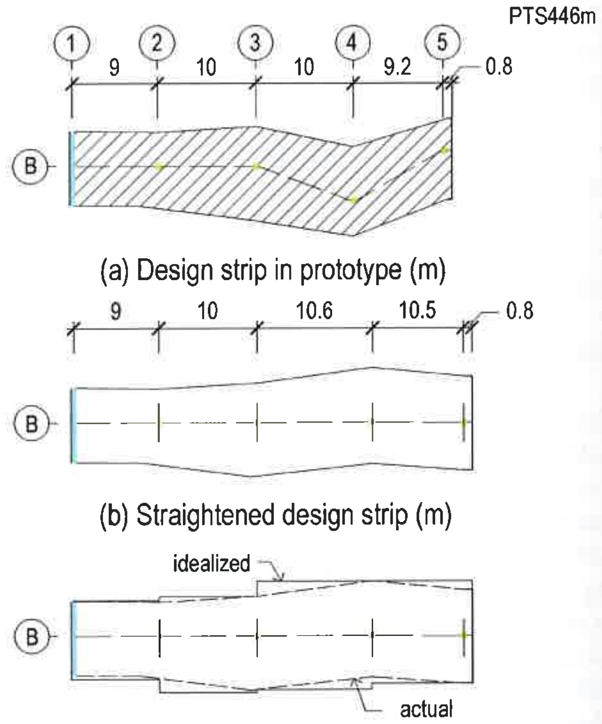
The breakdown of a floor into support lines, tributaries and design strips in two principal directions are explained in Chapter 3, as the first step in definition of load paths for design. The outcome is the subdivision of floor into design strips in each of the two orthogonal directions. In this example, we se-lect and complete the design of one of the design strips in X-direction. The remainder of the design strips will be treated in a similar manner.

The design strips in X-direction are shown in Fig. 1.3-1. Each design strip is extracted from the floor system and modeled in isolation as an idealized single design strip, such as the design strip for support line B shown in Fig. 1.3-2a.



Tributaries for Design Strips in X - Direction

FIGURE 1.3-1



Construction of Design Strip in Plan

FIGURE 1.3-2

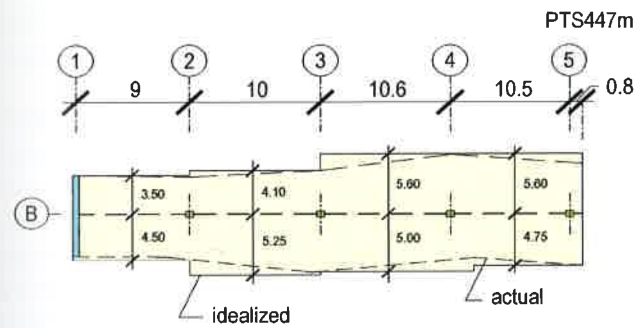
1.4 Idealized Design Strip

❖ Design Strip Dimensions

The extracted design is "straightened" to simplify analysis (Fig. 1.3-2b). The tributaries of each span of the extracted design strip are adjusted to the maximum width of the respective span on each side of the support line. The dimensions of the final design strip are shown in Figs. 1.4-1 and 1.4-2a.

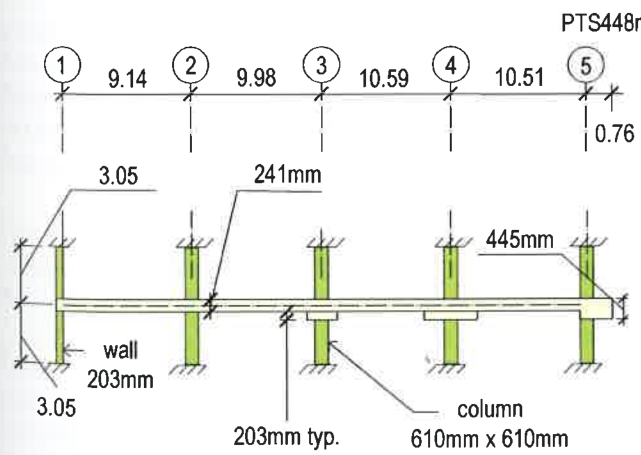
For gravity design of the structure, the practice in selection of boundary conditions of the extracted design strip is verbalized in ACI/IBC as follows. The strip is modeled with one level of supports immediately above and below the level under consideration. The far ends of the supports are assumed fixed against rotation.

The elevation of the idealized design strip and a three dimensional view of it are shown in Figs. 1.4-2, 1.4-3.



Plan of Design Strip B (m)

FIGURE 1.4-1



Design Strip in Elevation

FIGURE 1.4-2

❖ Section Properties

The section properties of each span are calculated using the gross cross-sectional area of the idealized design strip as shown in Figs. 1.4-1 and 1.4-2.

The stiffening of the slab due to the added thickness of the column drops and drop panels are accounted for in the calculation through their section properties. In SFM adopted in this example, the added stiffness in the slab immediately over the support is not included in the analysis. However, the EFM of analysis allows for the aforementioned increase in stiffness.

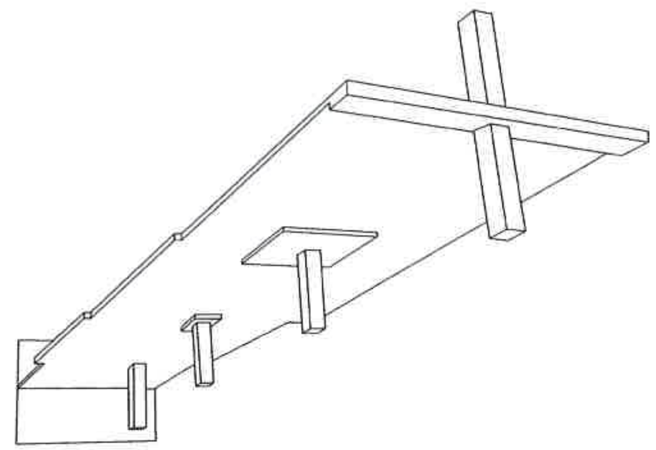


FIGURE 1.4-3 View of the Design Strip (P473)

2 - MATERIAL PROPERTIES

2.1 Concrete

f'_c, f_{ck} (28 day cylinder strength)[2] = 40 MPa
Weight = 24 kN/m ^{3}
Elastic Modulus $4700\sqrt{f'c}$ = 29725 MPa [ACI]
= 22* 10^{3*} [(f_{ck} +8)/ 10] $^{0.3}$ [3] [EC2, TR-43]
= 35220 MPa
Creep coefficient = 2
Material factor, γ_c = 1 [ACI]; 1.50 [EC2, TR-43]

The creep coefficient is used to estimate the long-term deflection of the slab.

2.2 Nonprestressed (Passive) Reinforcement

f_y = 460 MPa
Elastic Modulus = 200000 MPa
Material factor, γ_c = 1 [ACI]; 1.15 EC2
Strength reduction factor (bending), φ = 0.9 [ACI];
= 1 [EC2, TR-43]

2 Where cube strength is specified, the following conversion is used: cylinder strength = 0.8 times cube strength
3 EN 1992-1-1:2004(E) Table 3.1

TABLE 1.4-1 Section Property of the Design Strip (T1575I)

Span	Tributary Width (m)	Depth (mm)	I (mm⁴)	Y_b (mm)	Y_t (mm)
1	8.0	240	9.216e+9	120	120
2 Mid	9.35	240	1.077e+10	120	120
2 Right	9.35	440	2.458e+10	294	146
3 Left	10.6	440	2.620e+10	297	143
3 Mid	10.6	240	1.221e+10	120	120
3 Right	10.6	440	4.177e+10	271	169
4 Left	10.35	440	4.134e+10	271	169
4 Mid	10.35	240	1.192e+10	120	120
4 Right	10.35	440	7.347e+10	220	220
Cantilever	10.35	440	7.347e+10	220	220

Where

I = second moment of area of the tributary section; and

Y_t, Y_b = top and bottom distances of the centroid of the section to the extreme fibers respectively

2.3 Prestressing (Figs 2.3-1 through 2.3-3)
Material—low relaxation, seven wire ASTM 416 strand
Nominal strand diameter = 13 mm
Strand area = 99 mm²
Elastic Modulus = 200000 MPa
Ultimate strength of strand (f_{pu}) = 1860 MPa
Material factor, γ_c = 1 [ACI]; 1.15 [EC2, TR-43]

System
Unbonded System
Angular coefficient of friction (μ) = 0.07
Wobble coefficient of friction (K) = 0.003 rad/m
Anchor set (wedge draw-in) = 6 mm
Stressing force = 80% of specified ultimate strength
Effective stress after all losses[4] = 1200 MPa

Bonded System
Use flat ducts 20x80mm; 0.35 mm thick metal sheet housing up to five strands
Angular Coefficient of Friction (μ) = 0.2
Wobble Coefficient of Friction (K) = 0.003 rad/m
Anchor Set (Wedge Draw-in) = 6 mm
Offset of strand to duct centroid (z) = 3 mm
Effective stress after all losses = 1100 MPa

3 - LOADS

3.1 Selfweight
Slab=(240/1000)* 2400* 9.81/1000
= 5.65 kN/m²

3.2 Superimposed Dead Load
Superimposed dead load = 2.00 kN/m²
Total Dead Load = SW + SDL = 7.65 kN/m²
Span 1 DL = 7.65* 8.00 m = 61.20 kN/m
Span 2 DL = 7.65* 9.35 m = 71.53 kN/m
Span 3 DL = 7.65* 10.60 m = 81.09 kN/m
Span 4 DL = 7.65* 10.35 m = 79.18 kN/m
Added dead load due to column drop, drop panel and transverse beam:
Column drop DL (support 3)
= 0.2*1.5*2400*9.81/1000 = 7.06 kN/m
Load extends 0.75 m on each side of support 3)
Drop panel DL (support 4) =.2*3.6*2400*9.81/1000
= 16.95 kN/m (Load extends 1.8 m on each side of support 4)
Added beam depth (cantilever)
0.2*10.35*2400*9.81/1000 = 48.74 kN/m
(Load extends from 0.2 m left of support 5 to slab edge)

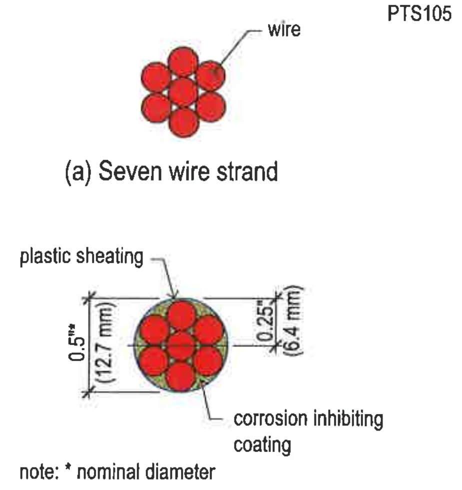
3.3 Live Load[5] : 3 kN/m²
Span 1 LL = 3* 8 = 24 kN/m
Span 2 LL = 3* 9.35 = 28.05 kN/m
Span 3 LL = 3* 10.60 = 31.80 kN/m
Span 4 LL = 3* 10.35 = 31.05 kN/m
Cantilever LL = 3* 10.35 = 31.05 kN/m
LL/DL ratio = 3/7.65 = 0.39
< 0.75 ∴ Do not skip live load

Live load is generally skipped (patterned), in order to maximize the design values. However, for two-way floor systems, ACI 318-11 does not require live load skipping,[6] provided the ratio of live to dead load does not exceed 0.75. In this example, as in most concrete floor systems for residential and office buildings, the ratio of live to dead load is less than 0.75. Hence, the live load will not be skipped.

The loading diagrams are shown in Fig. 3-1.

4 - DESIGN PARAMETERS

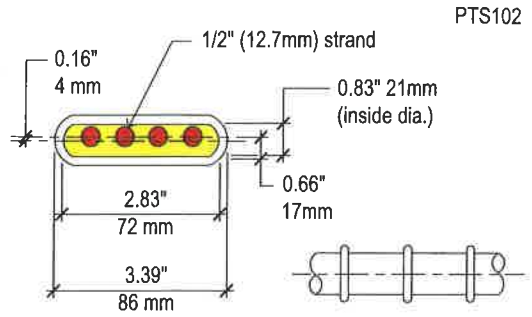
[4] For hand calculation, an effective stress of tendon is used. The effective stress is the average stress along the length of a tendon after all immediate and long-term losses. The value selected for effective stresses is a conservative estimate. When "effective stress" is used in design, the stressed lengths of tendons are kept short, as it is described later in the calculations.



(b) View of tendon

Unbonded Tendon

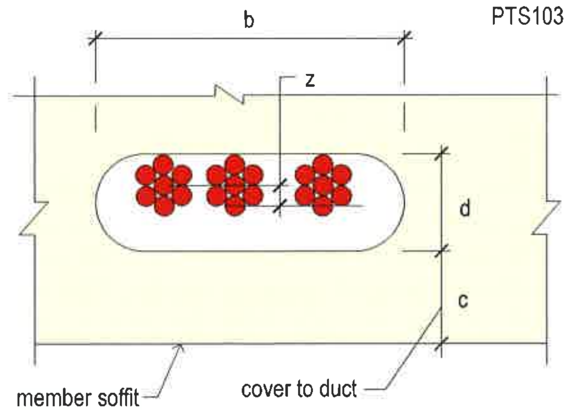
FIGURE 2.3-1 Section View of an Unbonded Tendon



(a) Section (b) Elevation

Example of a Grouted Flat Duct Used in Building Construction

FIGURE 2.3-2 Plastic Flat Corrugated Duct Grouted Tendon



b = duct width d = duct depth c= cover to duct

z = distance between center of strand and duct center

Section through a Flat Duct at Low Point

FIGURE 2.3-3

4.1 Applicable Codes
The design is carried out according to each of the following codes. Further, reference is made to the Committee Report TR-43, where appropriate.

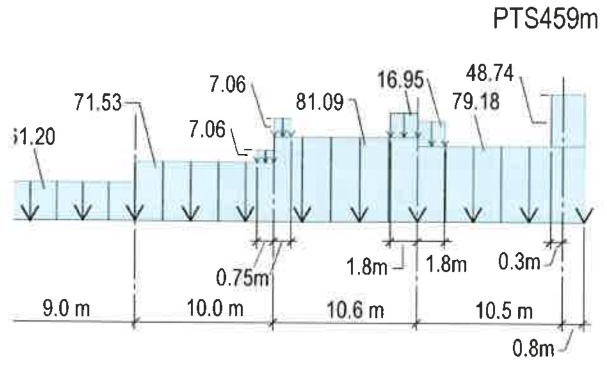
- ❖ IBC-2009 (ACI 318-2011)
- ❖ EC2(EN 1992-1-1:2004)

4.2 Cover to Rebar and Prestressing Strands
Minimum rebar cover = 20 mm top and bottom

Unbonded System
The slab is assumed to be in a non-corrosive environment. Cover to its reinforcement is based on a 2-hour fire rating with the exterior spans considered

[5] Live load for residential floors is generally 2 kN/m2. For commercial buildings it is somewhat more. Herein, conservatively 3kN/m2 is assumed. Live load is generally reduced based on the floor area it covers. Reduction of live load is described in IBC 2012 (Chapter 16). In this design example, live load is not reduced.

[6] ACI 318-11 (13.7.6)



(a) Dead load (kN/m)

(b) Live load (kN/m)

Loading on Design Strip

FIGURE 3.1

restrained. This requires a minimum cover of 20 mm, using IBC-12. Hence, the CGS (Center of Gravity of Strand) of 13 mm strand is 27 mm from top and bottom fibers of concrete outline. The existing concrete wall at one end of the design strip, and the down turned beam at the other end of it are considered adequate to provide restraint against in plane expansion of the slab for fire resistivity. Hence, the end spans are considered "restrained."[7]

Minimum strand cover = 20 mm
CGS. all spans = 27 mm

Bonded System
Minimum top and bottom rebar cover = 20 mm
For post-tensioning tendons: (Fig. 4.2-1)
Cover to duct = 20 mm
Distance to centroid of strand
= 20 + 10 + 3 = 33 mm
Where, 10mm is half duct diameter and z=3 mm
CGS. all spans = 33 mm

4.3 Allowable Stresses
A. Based on ACI 318-11/IBC 2009[8]

Allowable stresses in concrete are the same for bonded and unbounded PT systems

❖ For Sustained Load Condition
Compression = $0.45^* f'_c = 0.45^*40 = 18$ MPa
Tension = $0.5^* \sqrt{f'_c} = 3.16$ MPa

❖ For Total Load Condition
Compression = $0.60^* f'_c = 24$ MPa
Tension = $0.5^* \sqrt{f'_c} = 3.16$ MPa

❖ For Initial Condition (at Tranfer of Prestressing)
Compression = $0.60^* f'_{ci} = 0.6^* 30 = 18$ MPa
Tension = $0.25^* \sqrt{f'_c} = 1.58$ MPa

In ACI 318/IBC 2012 the allowable stresses for two-way systems and one-way systems are different. The values stated are for two-way systems. These values may not be exceeded. Using ACI-318, two-way systems are deemed to be essentially crack-free when in service. Cracking, if any is not of design significance.

B. Based on EC2[9]
EC2 does not specify "limiting" allowable stresses in the strict sense of the word. There are stress thresholds that trigger crack control. These are the same for both bonded and unbounded systems

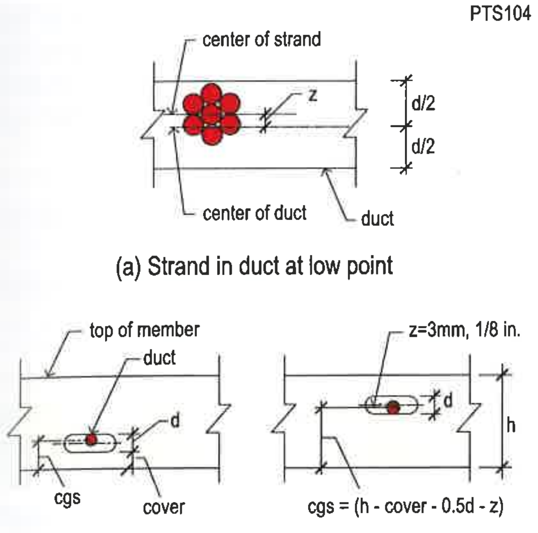
❖ For "Frequent" Load Condition
Concrete
Compression = $0.60^* f_{ck} = 0.6^* 40 = 24$ MPa
Tension (concrete) $F_t = f_{ct,eff} = f_{ctm}$[10]
$F_t = 0.30^* f_{ck}^{(2/3)} = 0.30^* 40^{(2/3)}$
= 3.51 MPa (Table 3.1, EC2)
Tension (non-prestressed steel) = $0.80^* f_{yk} = 0.8^*460$ = 368 MPa
Tension (prestressing steel) = $0.75^* f_{pk}$ = $0.75^* 1860 = 1395$ MPa

❖ For "Quasi-permanent" Load Condition
Compression = $0.45^* f_{ck} = 0.45^* 40 = 18$ MPa
Tension (concrete) = 3.51 MPa
same as frequent load combination

Unlike ACI 318/IBC, provisions in EC2 permit[11] overriding the allowable hypothetical tension stress in concrete, provided cracking is controlled not to exceed the selected "design crack width."

7 In IBC-12, where a span is free to expand in its own plane, it is considered "unrestrained," and is required to have a larger cover for fire resistivity than a span that is not free to expand (restrained). IBC Table 720.1
8 ACI 318-11, Sections 18.3

9 EN 1992-1-1:2004(E),Section 7.2
10 EN 1992-1-1:2004(E), Section 7.3.2(4)
11 EN 1992-1-1:2004(E), Section 7.3.2(4)



(a) Strand in duct at low point

(b) Tendon at low point (c) Tendon at high point

Position of Center of Gravity (cgs) of Strand at Extreme Positions in Member

FIGURE 4.2-1

❖ For "Initial" Load Condition (Table 3.1; EC2)
Tension (Unbonded) = $f_{ct,eff} = f_{ctm}$
$0.30^* f_{ci}^{(2/3)} = 0.30^* 30^{(2/3)} = 2.90$ MPa
Compression[12] = $0.60^* f_{ci} = 0.6^*30 = 18$ MPa

C. Based on TR-43[13] Using Panel Width
TR-43 Report provides two sets of allowable stresses. One is based on the traditional selection of design strips based on the full tributary and referred to as "full panel width," such as the one used in this design example and commercial software. The other set of allowable stresses is based on narrower design strips selected to more closely capture the local behavior of a slab. The latter, referred to as "design strip approach" is an option for processing solutions obtained from Finite Element analyses. For practical reasons, most engineers and automated commercial software tools use the "full panel width" option of allowable stresses, in particular, since both options are deemed to result in the same design.

For flat slabs, allowable stresses are the same for both bonded and unbonded systems, as well as for "frequent" and "quasi-permanent" load combinations.

There are two thresholds for hypothetical tension stresses. If the hypothetical tension stresses are below the first threshold, no bonded reinforcement need be added. If the hypothetical tension stresses exceed the first threshold, but are less than the second, a specified amount of bonded reinforcement must be provided. The hypothetical stresses are not permitted to exceed the upper threshold.[14] Grouted tendons can be considered as bonded reinforcement, as it is explained in greater detail in Section 7.4.

Tension (without bonded reinforcement)
For full panel[15] = $0.3\ f_{ctm,fl}$
$f_{ctm,fl}$ = larger of (1.6- h/1000) f_{ctm} or f_{ctm}[16]
= larger of (1.6 - 0.24) f_{ctm} or f_{ctm}
= larger of $1.36^* f_{ctm}$ or f_{ctm}
$f_{ctm} = 0.30^* f_{ck}^{(2/3)}$ (Table 3.1, EC2)
$= 0.30^* 40^{(2/3)} = 3.51$ MPa
Allowable lower threshold = $0.3^*1.36^* 3.51$
= 1.43 MPa

Tension (with bonded reinforcement)
For full panel = $0.9\ f_{ctm,fl}$
$= 0.9^*1.36^* 3.51 = 4.30$ MPa
Compression (support) = $0.3^* f_{ck} = 0.3^*40 = 12$ MPa
Compression (span) = $0.4^* f_{ck} = 0.4^*40 = 16$ MPa

❖ For "initial" load condition[17]
Tension = $0.4\ f_{ctm}$
where f_{ctm} refers to strength at stressing
$f_{ctm} = 0.30^* f_{ci}^{(2/3)} = 0.30^* 30^{(2/3)} = 2.90$ MPa
Allowable tension stress = $0.4^*2.90 = 1.16$ Mpa
Allowable compression stress = $0.40^* f_{ci} = 12$ Mpa

4.4 Crack Width Limitation
A. Based on ACI 318-11/IBC 2012
No explicit limit is imposed by the code for crack width calculation and or its control for two-way floor systems, since the designs are deemed to be essentially within the pre-cracking range of concrete.

B. Based on EC2
In EC2, the allowable crack width depends on whether the post-tensioning system used is "bonded," or "unbonded," and the load combination being considered.[18]

❖ Prestressed members with bonded tendons: 0.2 mm; to be checked for frequent load case.

12 EN 1992-1-1:2004(E), Section 5.10.2.2(5)
13 TR-43 Second Edition, Table 4. For tensile stress, stress limit without bonded reinforcement is considered.

14 The amount of bonded reinforcement to be added is explained in Section 7.4 "Minimum Reinforcement."
15 TR-43, 5.8.1 Table 3
16 EN 1992-1-1:2004(E),Eqn.3-23
17 TR-43 Second Edition, Table 5.
18 EN1992-1-1-2004 (E) Table 7.1N

❖ Prestressed members with unbonded tendons: 0.3 mm; to be checked at quasi-permanent load case.

C. Based on TR-43
For both prestressed systems[19] = 0.2 mm

4.5 Allowable Deflection

A. Based on ACI 318-11/IBC 2012[20]
In all major codes, the allowable deflection is tied to (i) the impact of the vertical displacement on occupants; (ii) the installed non-structural objects such as partitions, glass, or floor covering; and (iii) functional impairment, such as proper drainage. Details of the allowable values, their measurement and evaluation are given in reference [ADAPT TN292]. For perception of displacement by sensitive persons, consensus is limit of *L/250*, where L is the deflection span. It is important to note that this is the displacement that can be observed by a viewer.

The allowable values are:
Since in this design example carpet is assumed to be placed directly on the finished floor, the applicable vertical displacement is the total deflection subsequent to the removal of forms.

Total allowable deflection: L/240

The second deflection check is for potential damage to non-structural brittle construction, such as partitions, from displacement subsequent to installation of such members. The value recommended by ACI-318 is *L/480*. This is vertical displacement resulting from the full application of design live load together with the long-term deflection subsequent to the installation of construction likely to be damaged by deflection. Such installations are application of plaster on concrete masonry unit partitions or installation of dry wall (gypsum boards). Raw framing or masonry units that are not finished are not considered to be subject to the deflection limitations.

Total deflection subsequent to finish on partitions together with application of live load: L/480
Where, L is the length of deflection span. For this design example, the partitions are assumed to have been installed/finished 60 days after the floor is cast.

B. Based on EC2[21]
The interpretation and the magnitude of allowable deflections in EC2 are essentially the same as that of ACI 318. The impact of vertical displacement on the function of the installed members and the visual impact on occupants determine the allowable values. The following are suggested values:

Deflection subsequent to finishing of floors from Quasi-permanent combination: L/250
Deflection subsequent to installation of construction that can be damaged from load combination Quasi-permanent: L/500.

C. Based on TR-43[22]
TR-43 refers to EC2 for allowable deflections.

In summary, the allowable deflection from the two codes and the committee report are essentially the same. Conservatively, it can be summarized as follows:

Quasi Permanent Load Combination
Total deflection: L/250
Deflection subsequent to installation of construction that can be damaged: L/500

Brittle partitions are assumed to have been installed 60 days subsequent to date of casting the slab.

5 - ACTIONS DUE TO DEAD AND LIVE LOADS

Actions due to dead and live loads are calculated by a generic frame program, using the idealized frame dimensions shown in Fig. 5-1. The stiffness of each of the spans is based on the second moment of area given in Table 1.3-1. At locations of the column drop, drop panel, and transverse beam, the stiffness used includes the local thickening of the slab.

In Fig. 5-1 the column drop and drop panel are shown centered about the mid-depth of the slab, since it is assumed that most frame programs used by consultants. The shortcoming becomes critical when designing post-tensioned members, where the eccentricity of tendons with respect to that of the section is of central importance. Later in this design example, it will be illustrated how to account for this shortcoming, and obtain correct values with due allowance for eccentricities. The computer programs ADAPT-PT or ADAPT-Builder, automatically account for the shift in the centroid of a column drop/panel below that of the slab., These computer programs do not require an adjustment.

[19] TR-43 Second Edition, Section 5.8.3.
[20] ACI 318-11, Section 18.3.5
[21] EN 1992-1-1:2004(E), Section 7.4.1
[22] TR-43 Second Edition, Section 5.8.4

For hand calculations, a simple frame analysis is used (Simple Frame Method—SFM). The simple frame method of analysis lacks the specific features of the Equivalent Frame Method (EFM) as listed below:

(i) Increased stiffness of slab over slab/support interface is not accounted for. The stiffness of a slab over its support is assumed to be the same as that at the face of support;

(ii) Increased stiffness of the column within the slab, or within the column drop/panel is not accounted for. In other words, the stiffness of a column is assumed constant over its entire analysis length. Note that the analysis length of a column extends to the centroid of slab; and

(iii) The analysis does not account for the two-way action of the slab, as is implemented in the Equivalent Frame Method. The stiffness of the structure is strictly based on the cross-sectional geometry of the design strip.

The SFM is adequate when hand calculation is used for design. The EFM is more accurate, but it is too complex for hand calculation in the environment of a production oriented consulting office. It is important to note that the SFM provides a safe design, but not necessarily the most economical alternative. The EFM generally leads to smaller column moments, when compared to the SFM.

Examples of the EFM in the literature are generally limited to flat plates mostly without column drop or drop panel, and with uniform tributaries. The use of computer programs with EFM formulation is the practical way for design of complex floor systems with column drop, and/or drop panel, irregular tributaries and non-uniform loads.

The moments calculated from the frame analysis refer to the center line values. These are reduced to the face-of-support using the static equilibrium of each span. The computed moments from the analysis using Simple Frame Method (SFM) are shown in Fig. 5-2 and Fig.5-3. The values at each face-of-support and at midspan are summarized in Table 5-1.

The critical design moments are not generally at midspan. But, for hand calculation, the midspan is selected. The approximation is acceptable when spans and loads are relatively uniform.

6 - POST-TENSIONING

6.1 Selection of Design Parameters

Unlike conventionally reinforced slabs, where given geometry, boundary conditions, material properties and loads result in a unique design, for post-tensioned members in addition to the above a minimum of two other input assumptions are required, before a design can be concluded. A common practice is (i) to assume a level of precompression and (ii) target to balance a percentage of the structure's dead load. In this example, based on experience the level of precompression suggested is larger than the minimum required by ACI-318 code (0.86 MPa). Other major building codes do not specify a minimum precompression. Rather, they specify a minimum reinforcement. Use the following assumption to initiate the calculations.

1. Minimum average precompression = 1.00 MPa
2. Maximum average precompression = 2.00 MPa
3. Target Balanced Loading = 60% of total dead load, up to 80% where beneficial

The minimum precompression is used as the entry value (first trial) for design. The stipulation for a maximum precompression does not enter the hand calculation directly. It is stated as a guide for a not-to-exceed upper value. In many instances, floor slabs that require more than the maximum value stated can be re-designed more economically.

For deflection control the selfweight of the critical span is recommended to be balanced to a minimum

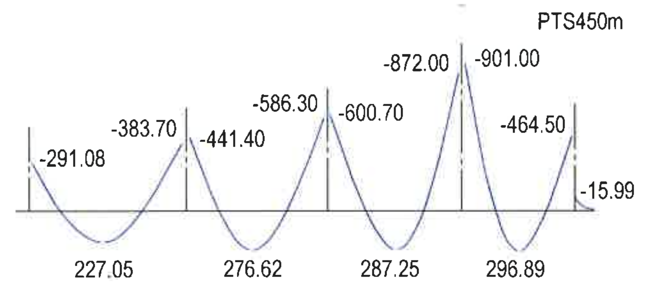
Table 5-1 Moments at Face-of-Support and Midspan (T158SI)

Span	Location	M_D (kN-m)	M_L (kN-m)
Span #1	Left FOS*	-291.08	-114.10
	Midspan	227.05	89.03
	Right FOS	-383.70	-150.50
Span #2	Left FOS	-441.40	-173.10
	Midspan	276.62	108.50
	Right FOS	-586.30	-229.60
Span #3	Left FOS	-600.70	-234.80
	Midspan	287.25	112.30
	Right FOS	-872.00	-335.70
Span #4	Left FOS	-901.00	-347.40
	Midspan	296.89	116.20
	Right FOS	-464.50	-180.90
Cantilever	Left FOS	-15.99	-3.88

* FOS = face-of-support

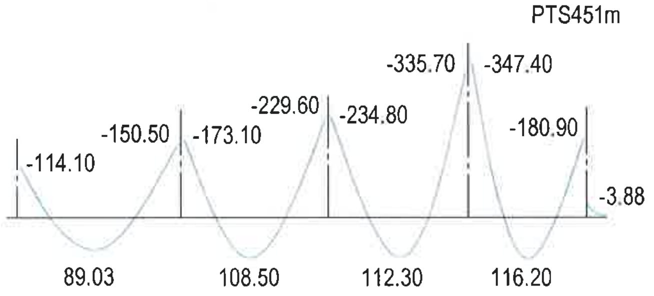
of 60%. Non-critical spans need not be balanced to the same extent.

Effective stress in prestressing strand
For unbonded tendons: f_{se} = 1200 MPa
For bonded tendons: f_{se} = 1100 MPa



Moments Due to Dead Loading (kNm)

FIGURE 5-1



Moments Due to Live Loading (kNm)

FIGURE 5-2

The design of a post-tensioned member can be based either on the "effective force", or the "tendon selection" procedure. In the effective force procedure, the average stress in a tendon after all losses is used in design. In this case, the design concludes with the total effective post-tensioning force required at each location. The total force arrived at the conclusion of design is then used to determine the number of strands required, with due allowance for friction and long-term losses. This provides an expeditious and simple design procedure for hand calculations. In the "tendon selection" procedure, the design is based on the number of strands with due allowance for the immediate and long-term losses. In the following, the "effective force" method is used to initiate the design. Once the design force is determined, it is converted to the number of strands required. A graphical presentation of the preceeding assumptions is given in Chapter 4, Fig 4.8.7.1-2.

The effective stress assumed in a strand is based on the statistical analysis of common floor slab dimensions for the following conditions (Fig. C6.1-1):

(i) Members have dimensions common in building construction;

(ii) Tendons equal or less than 38 m long stressed at one end. Tendons longer than 38m, but not exceeding 76m are stressed at both ends. Tendons longer than 76m are stressed at intermediate points to limit the unstressed lengths to 38m for one-end stressing or 76m for two-end stressing, whichever is applicable;

(iii) Strands used are the commonly available 13 or 15 mm nominal diameter with industry common friction coefficients as stated in material properties section of this design example; and

(iv) Tendons are stressed to $0.8f_{pu}$.

For other conditions, a lower effective stress is assumed, or tendons are stressed at intermediate points. In the current design, the total length of the tendon is 41 m. It is stressed at both ends. Detailed stress loss calculations, not included herein, indicate that the effective tendon stress is 1250 MPa for the unbonded system and also larger than assumed for the grouted system.

6.2 Selection of Post-Tensioning Tendon Force and Profile

The prestressing force in each span will be chosen to match a whole number of prestressing strands. The following values are used:

1. The effective force along the length of each tendon is assumed to be constant. It is the average of force distribution along a tendon.

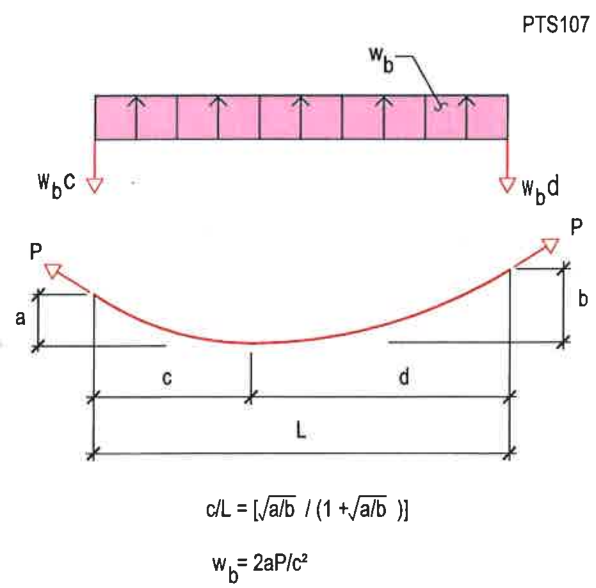
Unbonded tendons:
Force per tendon = 1200* 99 mm²/1000
= 118.8 ≈ 119.0 kN/ tendon
Use multiples of 119 kN when selecting the post-tensioning forces for design.

Bonded tendons:
Force per tendons = 1100* 99 mm²/1000
= 108.9 ≈ 109.0 kN/ tendon
Use multiples of 109 kN when selecting the post-tensioning forces for design.

2. Tendon profiles are chosen to be simple parabola.

These produce a uniform upward force in each span.

For ease of calculation the tendon profile in each span is chosen to be simple parabola from support centerline to support centerline (Fig. C6.2-1). The position of the low point is selected such as to generate a uniform upward force in each span. The relationship given in Fig. C6.2-1 defines the profile. For exterior spans, where the tendon high points are not generally at the same level, the resulting low point will not be at midspan. For interior spans, where tendon high points are the same, the low point will coincide with midspan. Obviously, the chosen profile is an approximation of the actual tendon layout used in construction. Sharp changes in curvature associated with the simple parabola profile assumed are impractical to achieve on site. The tendon profile at construction is likely to be closer to reversed parabola, for which the distribution of lateral tendon forces will be somewhat different as discussed henceforth. Tendon profiles in construction and the associated tendon forces are closer to the diagrams shown in Fig. C6.2-2 for two common cases.



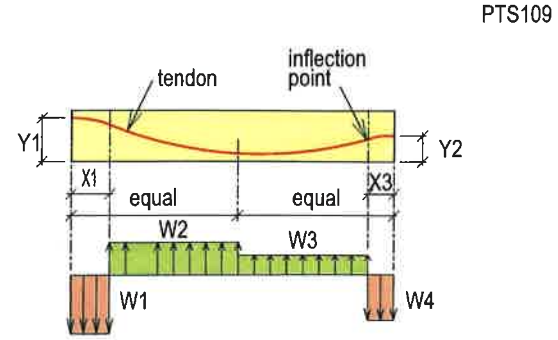
Geometry and Actions of a Parabolic Tendon

FIGURE C6.2-1

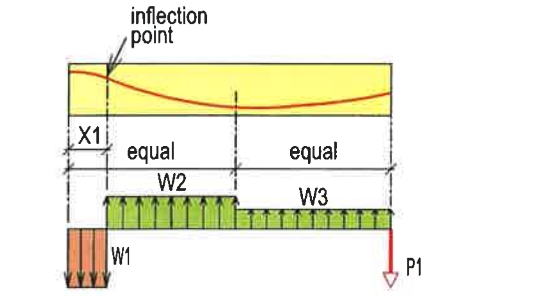
For the beam/cantilever at the right end, the profile selected is a straight line, due to short length of the overhang (Fig.C6.2-3).

6.3 Selection of Number of Strands

Determine the initial selection of number of strands for each span based on the assumed average precompression and the associated cross-sectional area of each span's tributary. Then, adjust the number of strands selected, based on the uplift they provide.

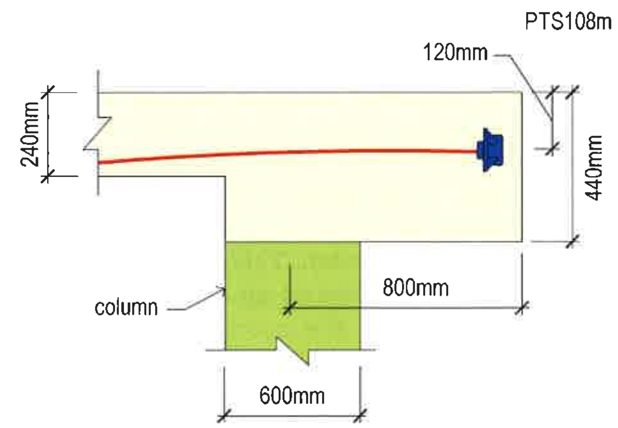


(a) Reversed parabola with two inflection points



(b) Reversed parabola with one inflection point

FIGURE 6.2-2 Two Examples of Common Tendon Profiles



Tendon Profile at Overhanhg

FIGURE C6.2-3 View of Overhang at Right End of Design Strip

Unbonded Tendon
Span 1 Area = 8.0 m* 1000* 240 mm = 1.92e+6 mm²

TABLE 6.3-1 Tendon Selection Based on Minimum Precompression (T1595I)

Span	Tributary (m)	Thickness (mm)	Area (mm²)	Force (kN)	Tendons	Selected tendons
1	8.00	240	1.920e+6	1920	17	20
2	9.35	240	2.244e+6	2244	20	20
3	10.6	240	2.544e+6	2544	23	23
4	10.35	240	2.484e+6	2484	22	23
Cant.	10.35	440	4.554e+6	4554	41	23

Span 1 Force = 1.0 MPa* 1.92e+6/1000 = 1920 kN
No. of Tendons = 1920/119.0 = 16.13; say 17
Calculated values for other spans are shown in table below

Bonded Tendon
Span 1 Area = 8.0 m* 1000* 240 mm = 1.92e+6 mm^2
Span 1 Force = 1.0 MPa* 1.92e+6/1000 = 1920 kN
No. of Tendons = 1920/109.0 = 17.61; say 18

It is noted that the number of strands required to satisfy the same criterion differs between the unbonded and bonded systems. Due to higher friction losses, when using bonded systems, more strands are generally needed to satisfy the in-service condition of design. For brevity, without compromising the process of calculation, in the following the same number of strands is selected for both systems.

The number of strands in Table 6.3-1 is based on a minimum precompression of 1.0 MPa at the midsection of each span. The added cross-sectional area of column drops, drop panels and transverse beams are disregarded in the calculation of the force for minimum precompression. The selected number of tendons is chosen to avoid an overly complicated tendon layout. Again, the precompression limit is disregarded for the cantilever, since the large value obtained is due to the depth of the beam having been used in the calculations, as opposed to slab depth.

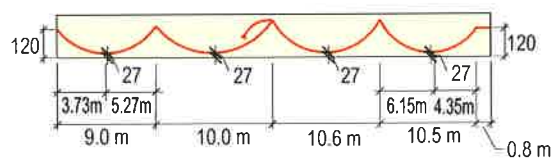
The tendon profile and force selected for unbonded tendons is shown in Fig. 6.3-1

6.4 Calculation of Balanced Loads
Balanced loads are the forces that a tendon exerts to its concrete container. It is generally broken down to forces normal to the centerline of the member (causing bending) and axial to it (causing uniform precompression) and added moments at locations of change in location of centroidal axis. Figure C6.2-2 shows two examples of balanced loading for members of uniform thickness.

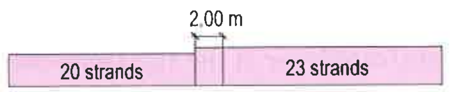
Span 1
Refer to Fig C6.2-1 and Fig. 6.4-1
a = 120—27 = 93 mm
b = 213—27 = 186 mm
L = 9.00 m
c = {[93/186]$^{0.5}$/[1 + (93/186)$^{0.5}$]}* 9.00 = 3.73 m
W$_b$/tendon = 2 P*a/c^2 = 119.0 kN* (2*93/1000)/3.73^2
= 119.0 kN/tendon* 0.013/m =1.59 kN/m/tendon
For 20 tendons W$_b$ = 1.59/ tendon* 20 tendons
= 31.8 kN/m
% of DL Balanced = 31.8/ 61.20 = 52%
(less than 60% target, but considered acceptable)
Balanced load reaction, left = 31.8 kN/m* 3.73
= 118.61 kN ↓
Balanced load reaction, right= 31.8 kN/m* 5.27
= 167.59 kN ↓

The profiles of the first and last spans are chosen such that the upward force on the structure due to the tendon is uniform. This is done by choosing the location of the tendon low point such that in each span the profile is a continuous parabola (Fig. C6.2-1). Both spans appear to be critical and will be designed for maximum drape, in order to utilize the maximum amount of balanced loading. If the low point of the tendon is not selected at the location determined by "c", two distinct parabolas result. Figure C6.2-2 illustrates the condition, where the low point is not at center of a tendon span.

PTS452m



(a) Assumed tendon profile (mm, UNO)

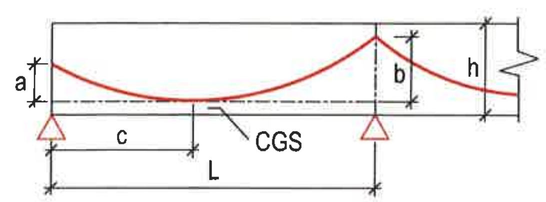


(b) Force diagram

Post Tensioning Profile and Force

FIGURE 6.3-1 Tendon Profile and Selected Force

PTS110



Tendon Elevation in First Span

FIGURE 6.4-1

Span 2
Span 2 has 20 continuous strands and three short strands (added tendons) that extend from span 3 to span 2 and terminate at its right end. The balanced load from each is calculated separately.
Continuous Tendons
a = 186 mm
L = 10.0 m

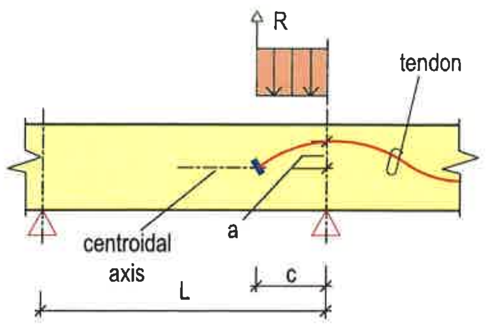
For a symmetrical parabola of span "L," drape "a," and uniform force "P," the force normal to L is given by $8P*a/L^2$.

W$_b$/ tendon = 8*P*a/ L^2 = (8*119*186/1000) /10^2
= 1.77 kN/m
For 20 tendons W$_b$ = 1.77* 20 tendons = 35.41 kN/m
% DL Balanced = 35.41/71.53 = 50% OK
Balanced load reaction:
Left = 35.41 kN/m* 5m = 177.05 kN ↓
Right = 35.41 kN/m* 5m = 177.05 kN ↓

Added Tendons

Increase in the number of strands from 20 to 23, from the third span on, results in 3 strands from the third span to terminate in the second span. The terminated three strands are dead-ended in the second span. The dead end is located at a distance 0.20*L from the right support, at the centroid of the design strip (Fig. C6.4-1). The tendons are assumed horizontal over the support and concave downward toward the dead end. Hence the vertical balanced loads of these tendons will be downward, with a concentrated upward force at the dead end.

PTS111



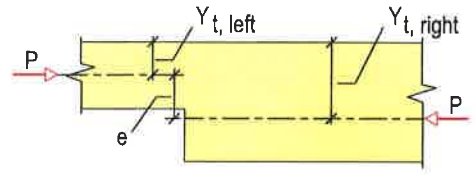
c = 0.2L ; $W_b = 2P\ a/c^2$ R = 2Pa/c

P=tendon force

Geometry and Actions of Tendons Terminated in Span

FIGURE C6.4-1

PTS112



Section at Change in Centroidal Axis

FIGURE C6.4-2

a = 93 mm
c = 0.20* 10 = 2.00 m
W$_b$ = (3* 119.0* 2* 93/1000)/2.0^2 16.60 kN/m ↓
Concentrated force at dead end = 16.60* 2.0
= 33.20 kN ↑
PT-induced Moments Due to Shift in Centroid

Because the centroid of the design strip section is shifted at the face of the column drop, drop panel and the edge beam, there will be a moment due to axial force from prestressing at each of these locations. These moments must be included in the balanced loading to obtain a complete and correct solution. The moments are simply the post-tensioning force in the section multiplied by the shift in the section's centroid (see Fig. 6.4-2).

Moment at Face of Column Drop
M = P* shift in centroid (e) = P* ($Y_{t\text{-Left}}$-$Y_{t\text{-Right}}$)

= 23* 119.0* (120-146)/1000 = -71.16 kN-m
M = 23* 119.0* (146-143)/1000 = 8.21 kN-m

Span 3
a = 186 mm
L = 10.60 m
W_b/ tendon = 8*P*a/ L^2 = (8*119.0*186/1000)/10.60^2
= 1.58 kN/m
For 23 tendons W_b = 1.58* 23 tendons = 36.34 kN/m
% DL Balanced = 36.34/81.09 = 45% ≈ 50% OK
Balanced load reaction:
Left = 36.34 kN/m* 5.3m = 192.60 kN ↓
Right = 36.34 kN/m* 5.3m = 192.60 kN ↓

PT-induced Moments Due to Shift in Centroid
Moment at face of left column drop:
M = P* shift in centroid
= 23* 119.0 kN* (143-120)/1000
= 62.95 kN-m
Moment at face of right drop panel:
M = P* ($Y_{t\text{-}Left}$-$Y_{t\text{-}Rigth}$)
= 23* 119.0* (120-169)/1000 = -134.11 kN-m
Moment at centerline of right support:
M = 23* 119.0* (169-169) ≈ 0 kN-m

Span 4
Refer to Figure C6.2-1
a = 120-27 = 93 mm
b = 213-27 = 186 mm
L = 10.50 m
C = {$[93/186]^{0.5}$/[1 + $(93/186)^{0.5}$]}* 10.5 = 4.35 m
Wb/ tendon = 119.0 kN* (2* 93/1000)/4.35^2
= 1.17 kN/m/tendon
For 23 tendons W_b = 1.17 kN/m/tendon* 23 tendons
= 26.90 kN/m
% DL Balanced = (26.90/79.18)*100 = 34 %

The dead load in the fourth span tends to produce an upward "lift" on adjacent spans. Since the fourth span is next to a somewhat larger, more heavily loaded third span, it is advantageous to design the fourth span with a lower level of balanced loading and allow its non-prestressing load to counteract the actions in the adjoining longer span. For this reason, the level of dead load balanced in the fourth span (34%) is acceptable, even though it is well below the target amount of 60% for the critical span. The above values will be assumed for a first try. If the stress check to follow will not be satisfactory the prestressing force will be adjusted.

Balanced Load Reaction
Left = 26.90* 6.15 = 165.44 kN ↓
Right = 26.9* 4.35 = 117.02 kN ↓
Moment at drop panel face
Left of span
M = 23* 119.0* (169-120)/1000 = 134.11 kN-m
Right of span
M = 23* 119.0* (120-220)/1000 =-273.70 kN-m

Cantilever
Tendon is horizontal and straight. Hence no upward force from tendon.
Moment due to dead end anchored away from centroid:
M = 23* 119.0* (220-120)/1000 = 273.70 kN-m

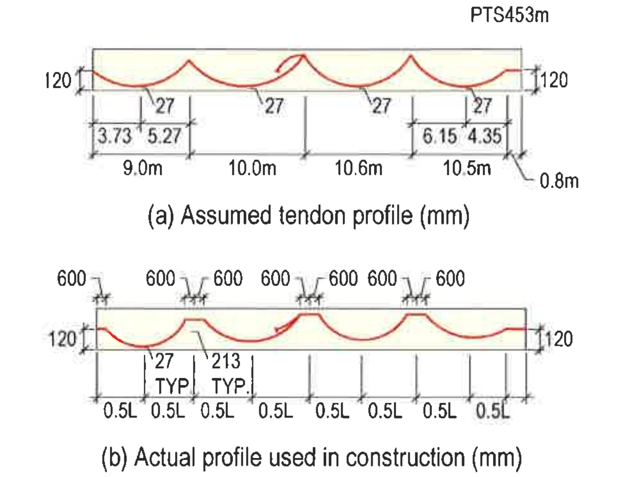
There is no vertical force over the length of the cantilever from the tendon profile of that span. However, the eccentricity of the tendon at edge of the slab results in a constant moment over the entire length of the cantilever.

The complete balanced loading consisting of up and down forces (part "a" of the figure) and the associated moments (part "b" of the figure) are shown in Fig. 6.4-4. In addition to the forces shown in the figure, there is an axial compressive force that is shown in Fig. 6.3-1b.

The actions shown in Fig. 6.4-3 represent the forces from the simplified tendon profile assumed for hand calculation and shown in Fig. C6.4-2a. In construction where unbonded system is uses, tendons in the design strip under consideration will be banded over the support line. In the perpendicular direction, the tendons will be distributed uniformly. The profile used for construction together with the one selected for hand calculation is shown in Fig. C6.4-2.

The forces exerted by a tendon to its container (concrete slab in this case) are always in static equilibrium, regardless of the geometry of tendon and the configuration of the member that contains the tendon. To guarantee a correct solution, it is critical to perform an equilibrium check for the balanced loads calculated (Fig. 6.4-3) before proceeding to the next step.

Equilibrium Check
Sum of forces in the vertical direction:
∑Forces ↑= -118.61 +(9*31.8)-167.59-177.05+(10*35.41)-(2*16.60)+33.20-177.05-192.60+(10.6*36.34)-192.60-165.44+ (10.5*26.90)-117.02 = 0.006 kN ≈ 0 OK
Sum of moments about the third support:
$\sum M_{3rd\ Support}$ = 118.61*19.0 + (167.59+177.05)*10.0-33.20*2.0- 31.8*9.0*14.5-35.41*10^2/2+16.60*2^2/2+36.34*10.6^2/2 +

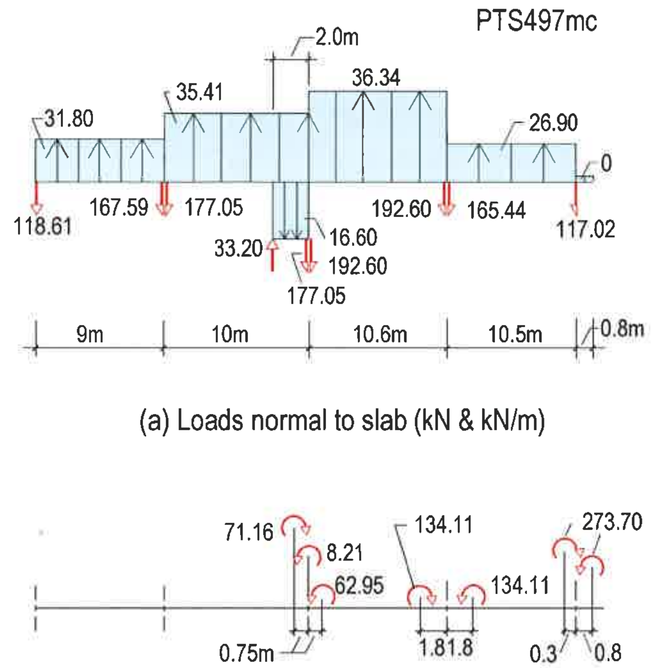


(a) Assumed tendon profile (mm)

(b) Actual profile used in construction (mm)

Presentation of Simplified and Actual Tendon Profiles

FIGURE C6.4-2



(a) Loads normal to slab (kN & kN/m)

(b) Moment (kN-m)

Balanced Loading

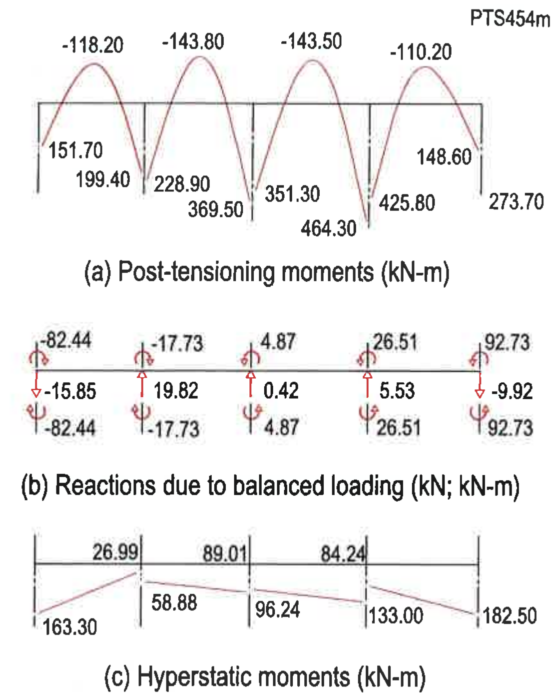
FIGURE 6.4-3

26.90*10.5*15.85-(192.60+ 165.44)*10.6-117.02*21.1-71.16+8.21+134.11+62.95-134.11-273.70+273.70 = 0.46 ≈ 0 kN-m OK

6.5 Determination of Actions due to Balanced (Post-Tensioning) Loads

The distributions of post-tensioning moments due to balanced loading, and the corresponding reactions at the slab/support connections, are shown in Fig. 6.5-1. These actions are obtained by applying the balanced loads shown in Fig. 6.4-3 to the frame shown in Fig. 5 1.

Actions due to post-tensioning are calculated using a standard frame program. The input geometry and boundary conditions to the standard frame program are the same as used for the dead and live loads.



(a) Post-tensioning moments (kN-m)

(b) Reactions due to balanced loading (kN; kN-m)

(c) Hyperstatic moments (kN-m)

Post-Tensioning Actions on Design Strip

FIGURE 6.5-1

7 CODE CHECK FOR SERVICEABILITY

7.1 Load Combinations

The following lists the recommended load combinations of the building codes covered for serviceability limit state (SLS).

❖ ACI, IBC
Total load condition 1*DL + 1*LL + 1*PT
Sustained load condition 1*DL + 0.3*LL + 1*PT[23]

[23] ACI-318 specifies a "sustained" load case, but does not stipulate the fraction of live load to be considered "sustained." It is left to the judgment of the design engineer to determine the applicable fraction. The fraction selected varies between 0.2 and 0.5. The most commonly used fraction is 0.3, as it is adopted in this design example.

❖ EC2, TR43
Frequent load condition 1*DL + 0.5*LL + 1*PT
Quasi-permanent load condition 1*DL + 0.3*LL + 1*PT

For serviceability, the actions from the balanced loads due post-tensioning (PT) are used. The background for this is explained in detail in reference [Aalami, 1990].

7.2 Stress Check

For hand calculation, the critical locations for stress check are selected based on engineering judgment. The selected locations may or may not coincide with the locations of maximum stress levels. This will introduce a certain degree of approximation in design, which reflects the common practice for hand calculations. Computer solutions generally calculate stresses at multiple locations along a span, thus providing greater accuracy. For brevity, only three locations will be selected for this design example. Point *A* is at the face-of-support; Point *B* is at the face of the drop panel; and point *C* is at the midspan (Fig. 7.2-1).

Using the Moment diagrams of Fig. 5-2 and 5-3 as guide, several critical locations are identified for the stress check. These are shown as sections A, B and C in Fig. 7.2-1.

Stresses
$\sigma = (M_D + M_L + M_{PT})/S + P/A$
$S = I/Y_c$

Where, M_D, M_L, and M_{PT} are the moments across the entire tributary of the design strip. S is the section modulus of the entire tributary; *A* is the cross-sectional area of the entire tributary; *I* is the second moment of area of the entire tributary; and Y_c is the distance of the centroid of the entire tributary to the farthest tension fiber of the entire tributary.

The parameters for stress check at point A are:
S_{top} = 4.134e+10/169 = 2.446e+8 mm^3
S_{bot} = 4.134e+10/271 = 1.525e+8 mm^3
A = 10.35* 1000* 240 + 3600* 200
= 3.204e+6 mm^2
P/A = -2737*1000/3.204e+6 = -0.85 MPa

A. Based on ACI 318-11/IBC 2012

Stress checks are performed for the two load conditions of total load and sustained loads.

At Point A

❖ Total Load Combination
Stress limit in compression: 0.60* 40 = 24 MPa
Stress limit in tension: 0.5*√40 = 3.16 MPa
$M_D + M_L + M_{PT}$ = (-901-347.40 + 425.80)
= -822.60 kN-m

Bottom fiber
σ = -822.60* 1000²/1.525e+8-0.85 = -6.24 MPa Compression < -24 MPa OK

Top Fiber
σ = 822.60*1000²/2.446e+8-0.85 Mpa = 2.51 MPa Tension < 3.16 MPa OK

❖ Sustained Load Combination
Stress limit in compression: 0.45* 40 = 18 MPa
Stress limit in tension: 0.5*√40 = 3.16 MPa
$M_D + 0.3\ M_L + M_{PT}$ = (-901-0.3*347.40 + 425.80)
= -579.42 kN-m

Bottom Fiber
σ = -579.42* 1000²/1.525e+8-0.85 = -4.65 MPa Compression< -18 MPa OK

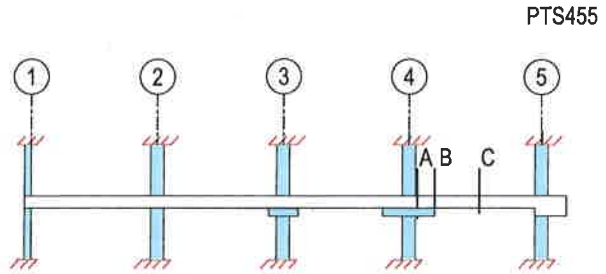


FIGURE 7.2-1

Top Fiber
σ = 579.42* 1000²/2.446e+8-0.85 = 1.52 MPa Tension < 3.16 MPa OK

B. Based on EC2

Stress checks are performed for the two load conditions of frequent load and quasi-permanent loads.

❖ Frequent Load Condition
$\sigma = (M_D + 0.5\ M_L + M_{PT})/S + P/A$

At Point A
Stress Thresholds
Compression = 0.60* 40 = -24 Mpa
Tension = f_{ctm} = 3.51 MPa
$M_D + 0.5M_L + M_{PT}$
= (-901-0.5*347.40 + 425.80)= -648.90 kN-m

Top Fiber
σ = 648.90*1000²/2.446e+8-0.85 = 1.80 MPa Tension < 3.51 MPa OK
Bottom Fiber
σ = -648.90* 1000²/1.525e+8-0.85 =-5.10 MPa Compression <-24 MPa OK

❖ Quasi-permanent load condition:
$\sigma = (M_D + 0.3\ M_L + M_{PT})/S + P/A$

At Point A
Stress Thresholds
Compression = 0.45* 40 = -18 MPa
Tension = f_{ctm} =3.51 MPa
$M_D + 0.3M_L + M_{PT}$
= (-901-0.3*347.40 + 425.80) = -579.42 kN-m

Top Fiber
σ = 579.42*1000²/2.446e+8-0.85 = 1.52 MPa Tension < 3.51 MPa OK

Bottom Fiber
σ = -579.42* 1000²/1.525e+8-0.85 = -4.65 MPa Compression <-18 MPa OK

C. Based on TR-43

❖ Frequent load condition:
$\sigma = (M_D + 0.5\ M_L + M_{PT})/S + P/A$

At Point A
Stress Limits
Compression (support) = 0.3* f_{ck} = 0.3*40 = 12 MPa
Tension (without bonded reinforcement)
= 0.3 $f_{ctm,fl}$ = 0.3*1.36*3.51 = 1.43 MPa
Tension (with bonded reinforcement) = 0.9 $f_{ctm,fl}$
= 0.9*1.36*3.51 = 4.30 MPa
$M_D + 0.5M_L + M_{PT}$ = (-901-0.5*347.40 + 425.80)
= -648.90 kN-m

Top fiber
σ = 648.90*10002/2.446e+8-0.85 MPa = 1.80 MPa Tension > 1.43 MPa
but less than 4.30 MPa; hence bonded reinforcement required.[24]

Bottom fiber
σ = -648.90* 1000²/1.525e+8-0.85 = -5.10 MPa Compression <-12 MPa OK

Other points are evaluated in a similar manner. The outcome is listed in the following table (Table 7.2-1):

[24] The required bonded reinforcement is calculated in Section 7.4-Minimum Reinforcement

The following illustrates the calculation of moments at interior of a span, such as point B for span under consideration.

Centerline moments and shears for *DL*, *LL* and *PT* obtained from frame analysis, along with the externally applied loads are shown below for the fourth span. The calculation of the values at the face-of-support follows simple statics of the free-body diagram shown below. In the following the calculation of moment at the face of drop panel in the fourth span is detailed. Other locations follow a similar procedure (Fig. 7.2-2).

Moment due to DL at the face of drop panel distance 1.80m from the fourth support

M_{DL} = 388.16* 8.7-575.16-48.72*0.3*8.55-79.18* 8.7²/2 = -319.70 kN-m
Moment due to LL
M_{LL} = 146.19* 8.7-223.37-31.05* 8.7²/2
= -126.60 kN-m
Moment due to PT
M_{PT} = -126.80* 8.7 +459.17+ 26.90* 8.7²/2-273.70+134.11 = 234.45 kN-m

7.3 Crack Width Control

A. Based on ACI 318-11/IBC 2012

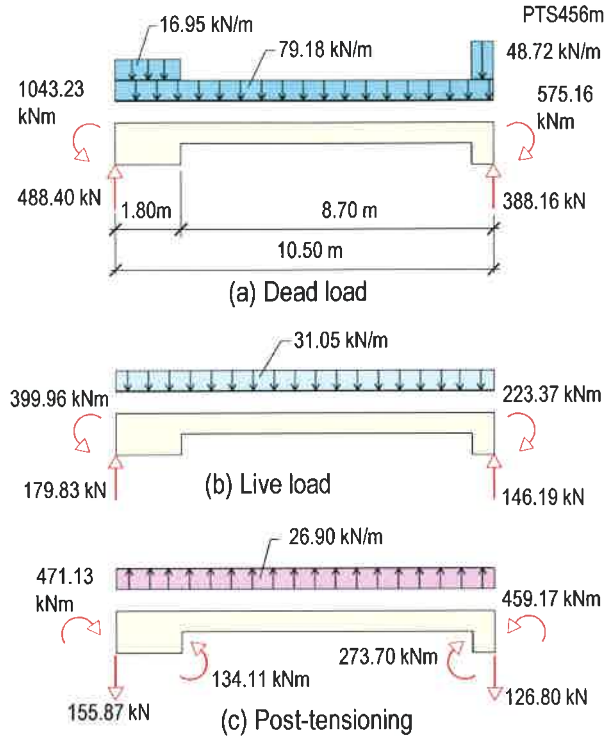
ACI 318-11/IBC 2012 do not stipulate specific measures to follow for crack control of slabs designed as two-way systems. The limit imposed on tensile stresses keeps the slabs essentially crack free, when in service.

B. Based on EC2 and TR-43[25]

The allowable crack width for members reinforced with unbonded tendons (Quasi-permanent load combination) is 0.3 mm, and for bonded tendon (Frequent load combination) is 0.2 mm. Since in this example the maximum computed tensile stress is within the threshold limit, crack width calculation is not required. If the computed tensile stress exceeds the threshold, EC2 recommends to limit the bar diameter and bar spacing to the values given in Table 7.2N or 7.3N of EC2 to control the width of probable cracks. The following example illustrates the point.

EXAMPLE
To illustrate the procedure for crack control recommended in EC2, as an example let the maximum tensile stress exceed the threshold value by a large margin.

[25] EN 1992-1-1:2004(E) , Section 7.3.3, and TR-43 2nd Edition, Section 5.8.3



Free Body Diagram of Fourth Span

FIGURE 7.2-2

Given: computed hypothetical farthest fiber tensile stress in concrete f = 30MPa
Required : reinforcement design for crack control

Calculate stress in steel at location of maximum concrete stress: σ_s = (f/Ec)*Es
Where f is the hypothetical tensile stress in concrete under service condition
σ_s = (30/35220)*200000 = 170 MPa (this is a hypothetical value)
Crack spacing can be limited by either restricting the bar diameter and/or bar spacing. Use the maximum bar spacing from Table 7.3 N for the σ_s of 170 MPa.
From Table: for 160 MPa-300 mm
200 MPa-250 mm
By interpolation, maximum spacing for 170 MPa is 287 mm.
Limit the spacing of reinforcement to 285 mm or less (280 mm) in order to control cracking. Note that based on the magnitude of the computed tensile stress in concrete the area of the required reinforcement is calculated separately,

7.4 Minimum Reinforcement

There are several reasons why the building codes specify a minimum reinforcement for prestressed members. These are:

❖ **Crack control, where potential of cracking exists:** Bonded reinforcement contributes in mitigating local cracks. The contribution of bonded reinforcement to crack control is gauged by the stress it develops under service load. Change of stress in bonded reinforcement from applied strain is a function of its modulus of elasticity and its cross-sectional area. Hence, the area of reinforcement considered available for crack control is *(As + Aps)*, where *Aps* is the area of bonded tendons. It is recognized that both bonded and unbonded prestressing provide precompression. While the physical presence of an unbonded tendon may not contribute to crack control, the contribution through the precompression it provides does. However, for code compliance and conformance with practice, the contribution of unbonded tendons is not included in the aforementioned sum.

❖ **Ductility:** One of reason ACI-318 specifies a minimum bonded reinforcement over supports of members reinforced with unbonded tendons is to provide ductility at the location. Where unbonded tendons are used, the required minimum area is provided through As only. Current ACI 318/IBC do not specify a minimum of non-stressed bonded reinforcement in post-tensioned members reinforced with bonded tendons.

Use 16 mm bars (Area = 201 mm^2; Diameter = 16 mm) for top and bottom, where required
d = 240- 20-16/2 =212 mm

A. Based on ACI 318-11/IBC 2012[26]
❖ Unbonded Tendons
Supports

ACI 318[27]/IBC require a minimum area of passive (non-stressed reinforcement to be placed over the supports, where unbonded tendons are used. The minimum area is expressed in terms of the cross-sectional geometry of the design strip, and the strip orthogonal to it. A_{cf} is the larger gross cross-sectional area of the design strips in the two orthogonal directions for the support under consideration. Figure C7.4-1 illustrates the applicable locations to

[26] ACI 318-11, Section 18.9
[27] ACI 318-11, Section 18.9.3

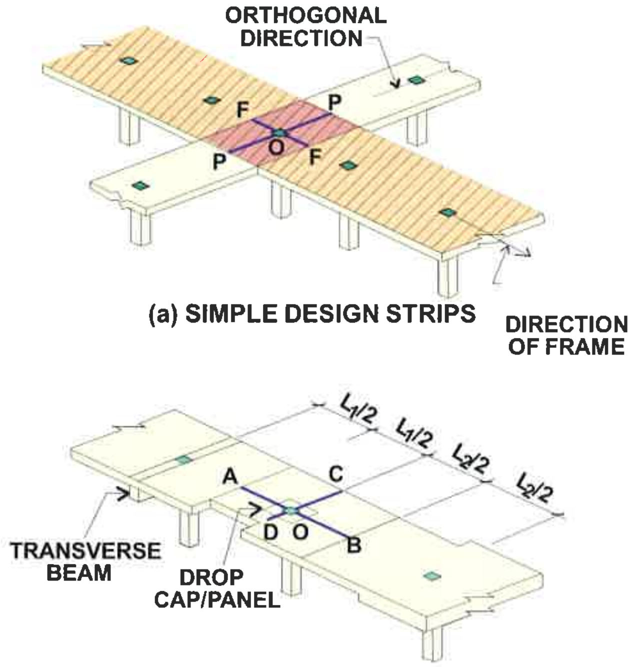
determine the cross-sectional areas. Line *PP* refers to the section in the design strip direction and *FF* to the section orthogonal to it.

A_s = 0.00075* Acf
At section A (Fig. 7.2-1):
In direction of design strip:
A_s = 0.00075* 0.5*(10600*240 +10500*240)
= 1899 mm^2
In the orthogonal direction to the design strip the spans adjacent to the support under consideration are 10.60 and 10.35 m. Hence,
A_s = 0.00075* 0.5*(10600*240 +10350*240)
= 1886 mm^2
A_s = 1899 mm^2 applies
Number of bars = 1899/201 = 9.4
Use 10-16mm bars = 10* 201 mm2 = 2010 > 1899 mm^2 provided top
Spans

The minimum passive reinforcement at midspan for unbonded tendons depends on the value of computed (hypothetical) tension at the bottom fiber. If the hypothetical tension stress is less than $0.166\,f'^{0.5}_c$, based on ACI 318,[28] no midspan minimum bottom rebar is required. It is re-iterated that the computed tensile stress is not permitted to exceed $0.5\,f'^{0.5}_c$.

At Point C in Span
At midspan A_s = N_c/(0.5*f_y) if hypothetical tensile stress > 0.166*$\sqrt{f'_c}$
where N_c is the total of tension force in the tensile zone of the section
Computed hypothetical tensile stress: f_{ct} = 1.95MPa
Stress Limit = 0.166* √40 = 1.05 MPa
1.95 MPa > 1.05 MPa ∴ Minimum steel is required
Compressive stress at top: fc = - 4.15 MPa
The relationships given in Fig. 7.4-1 will be used to determine the force of tensile zone (N_c)
Depth of tension zone from bottom
= 1.95* 240/(1.95+4.15) = 77 mm
N_c = 77mm* 1.95 MPa* 10350/(2* 1000)
= 777.03 kN
A_s = 777.03*1000 /(0.5*460) = 3378 mm^2
Number of bars = 3378/ 201 =16.8
Use 17-16 mm bars = 17* 201 = 3417 > 3378 mm^2 OK

❖ Bonded (Grouted) Tendons
There is no requirements for minimum reinforcement based on either geometry of the design strip, nor its



(b) EXAMPLE OF A PRACTICAL DESIGN STRIP

FIGURE C7.4-1 Illustration of Sections for Minimum Rebar (P563)

PTS114

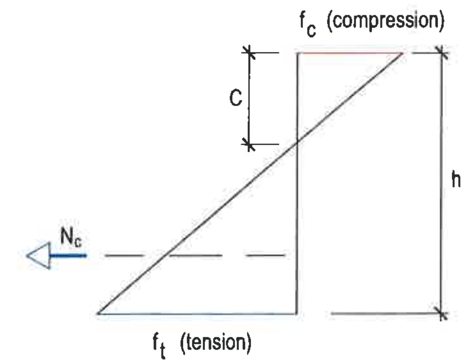


FIGURE 7.4-1 Distribution of Strain Over Section of Member

hypothetical tensile stresses. The minimum requirement is handled through the relationship between the cracking moment of a section and its nominal strength in bending. This is handled in the "strength" check of the member.

B. Based on EC2[29]
EC2 specifies the same requirement for minimum reinforcement at supports and spans, and also for both unbonded and bonded tendons. Two checks apply. One is based on the cross-sectional geometry of

[28] ACI 318-11 Section 18.9.3.2

[29] EN 1992-1-1:2004(E), Section 9.3.1 & 7.3.2

TABLE 7.2-1 Summary of Service Stress Checks (T160SI)

Span 4		Section A	Section B	Section C
Based on ACI 318-11/IBC 2009				
Sustained load	f_t (MPa)	1.52	0.14	-3.33
	f_b (MPa)	-4.65	-2.34	1.13
	F_t (MPa)	3.16	3.16	-18
	F_b (MPa)	-18	-18	NA
		OK	OK	OK
Total load	f_t (MPa)	2.51	1.03	-4.15
	f_b (MPa)	-6.24	-3.23	1.95
	F_t (MPa)	3.16	3.16	-24
	F_b (MPa)	-24	-24	3.16
		OK	OK	OK
Based on EC2				
Frequent Load	f_t (MPa)	1.8	0.4	-3.56
	f_b (MPa)	-5.1	-2.6	1.36
	F_t (MPa)	3.51	3.51	-24
	F_b (MPa)	-24	-24	3.51
		OK	OK	OK
Quasi-Permanent Load	f_t (MPa)	1.52	0.14	-3.33
	f_b (MPa)	-4.65	-2.34	1.13
	F_t (MPa)	3.51	3.51	-18
	F_b (MPa)	-18	-18	3.51
		OK	OK	OK
Based on TR-43				
Frequent Load	f_t (MPa)	1.8	0.4	-3.56
	f_b (MPa)	-5.1	-2.6	1.36
	F_t (MPa)	4.3	4.3	-16
	F_b (MPa)	-12	-12	4.3
		OK	OK	OK

Note: F_t and F_b are the respective top and bottom fiber allowable (threshold) stresses; F_c is allowable compressive stress.

the design strip and its material properties and the other on computed stresses. In the former, the minimum reinforcement applies to the combined contributions of stressed and non-stressed reinforcement. Hence, the participation of each is based according to the strength it provides, the prestressing steel is accounted for with higher values. The reinforcement requirement for crack control is handled separately.

❖ Unbonded and Bonded Tendons

Supports

At section A (Fig. 7.2-1):

$A_{smin} \geq (0.26^* f_{ctm}{}^*b_t{}^*d/f_{yk}) \geq 0.0013^* b_t{}^*d$

Since in EC2 the minimum reinforcement is a function of (bt^*d) cross-sectional area, at the face-of-support the cross-sectional area including the drop panel is used.

Cross-sectional Area

$b_t = 10350$ mm

Drop panel width = 3,600 mm

Drop panel depth below slab = 200 mm

Tributary cross-sectional area = 10,350*240 + 3,600*200 = $3.204{}^*10^6$ mm²

$f_{ctm} = 0.3^*40(2/3) = 3.51$ MPa

(i) $A_{smin} = 0.26^* f_{ctm}{}^*b_t{}^* d/f_{yk}$

$= 0.26^* 3.51^* 3.204^*10^6/460 = 6{,}356\ mm^2$

(ii) $A_{smin} = 0.0013^* b_t{}^* d$

$= 0.0013^* 3.204^* 10^6 = 4{,}165.2\ mm^2$

Therefore, $A_{smin} = 6{,}356\ mm^2$

Contribution of reinforcement from bonded Prestressing:

Aps*(fpk/fyk) = 23* 99*1860/460

= 9207 mm² > 6356 mm²

Hence, no additional bonded reinforcement is required.

Span

At section C in span (Fig. 7.2-1)

$b_t = 10350$ mm

(i) $A_{smin} = 0.26^* f_{ctm}{}^*b_t{}^*d/f_{yk}$

$= 0.26^*3.51^* 10350^*212/460 = 4353$ mm2

(ii) $A_{smin} = 0.0013^*b_t{}^*d$

$= 0.0013^*10350^* 212 = 2852\ mm^2$

Hence, $A_{smin} = 4353\ mm^2$

Contribution of reinforcement from bonded Prestressing:

Aps*(fpk/fyk) =

23* 99*1860/460 =9207 mm² > 4353 mm²

Hence, no additional bonded reinforcement is required.

❖ Minimum Reinforcement for Crack Control

In EC2 necessity of reinforcement for crack control is triggered, where computed tensile stresses exceed a code-specified threshold.

At all the three locations selected for code compliance, the hypothetical tensile stress of concrete is below the threshold for crack control. Hence, no crack control reinforcement is required.

EXAMPLE

For demonstration of EC2[30] procedure for crack control, let the maximum hypothetical tensile stress in concrete exceed the threshold set in the code (3.51MPa). Determine the required crack control reinforcement for the section reinforced with unbonded tendons.[31]

Given

$f_b = 3.7$ MPa (tension) at bottom

$f_t = -5.2$ MPa (compression) at top

Depth of section = 240 mm

Width of section = 10,350 mm

[30] EN 1992-1-1:2004(E), Section 7.3.2(3)

[31] For members reinforced with grouted tendons, the cross-sectional area of grouted tendons can be used to contribute to the minimum required area for crack control.

Required: Reinforcement for Crack Control

$\sigma_s = f_{yk} = 460$ MPa

k = 1

Depth of tension zone at bottom, using Fig. 7.4-1

= 3.7*240/(3.7 + 5.2) = 100 mm

$A_{ct} = 100^*10350 = 10.35e{+}5\ mm^2$

$k_c = 0.4^* [1-(\sigma_c /(k1\ (h/h^*)\ f_{ct,eff}))]$

$\sigma_c = N_{ED}/bh = 1.10$ MPa (average precompression)

$h^* = h = 240$ mm

$k_1 = 1.5$ (since section is in compression)

Criteria

$f_{ct,eff} = f_{ctm} = 0.3^*(40)^{(2/3)} = 3.51$ MPa

Design

$k_c = 0.4^* [1-(1.10 /(1.5\ (240/240)\ 3.51)] = 0.32$

$A_{smin} = k_c\ k\ f_{ct,eff}\ A_{ct} / \sigma_s$

$A_{smin} = 0.32^* 1^* 3.51^* 10.35e{+}5 /460 = 2499\ mm^2$

C. Based on TR-43

If the hypothetical tensile stress calculated for a panel (design strip as used in this example) exceeds the specified threshold given below, add non-prestressed rebar in addition to the prestressing to resist *Nc*[32]

(i) where unbonded tendons are used, and the hypothetical full tributary tensile stress exceeds $0.3\ f_{ctm,fl}$; and

(ii) where bonded tendons are used, and the hypothetical full tributary tensile stress exceeds $0.9\ f_{ctm,fl}$.

The amount of non-tensioned reinforcement depends on the tensile force (*Nc*) developed in the tensile zone of the location being considered. The area of (*As + Aps*) shall be adequate to resist Nc, where Aps is the area of available "bonded" reinforcement.

❖ Unbonded Tendons

TR43 specifies a minimum amount of non-prestressed reinforcement over the supports. The required minimum is based on both the cross-sectional geometry of the design strip and the computed tensile stresses.

At Support

Based on Geometry[33]

$A_{smin} = 0.00075$ Acf

Acf = cross-sectional area of the design strip in direction of analysis

[32] TR-43 2nd Edition, Section 5.8.1; Table 4

[33] TR-43 2nd Edition, Section 5.8.8

A_s = 0.00075* 0.5*(10600*240 +10500*240)
= 1899 mm^2

Based on Computed Stresses[34]
Refer to Fig. 7.4-2 where f_{cc} is the concrete fiber stress in compression; f_{ct} is the extreme concrete fiber stress in tension.
Depth of tension zone: h-x = -f_{ct}*h/(f_{cc}-f_{ct})
f_{ct} = tensile stress = 1.80 MPa
f_{cc} = compressive stress =-5.10 MPa
h-x = 1.80*440/(5.10+1.80) = 115 mm
As = F_t /(5*f_{yk} /8)
Where F_t is the total tensile force over the tensile zone of the entire section
F_t = -f_{ct}* b* (h-x)/2
= 1.80*10350*115/(2*1000) = 1071.23 kN
As = 1071.23*1000/(5*460/8) =3726 mm^2
The applicable rebar for this condition is the calculated value less area of unbonded tendons. Hence
As = 3726-23*99 = 1,449 mm^2
Comparing (i) and (ii), As = 1,899 mm^2
Use 10-16mm bars = 10* 201 mm^2
= 2010 > 1,899 mm^2 OK

At Span
At span bonded reinforcement is required if:
computed stress is greater than 0.3 $f_{ctm,fl}$ = 1.43 MPa.
f_{ct} =calculated tensile stress = 1.36 MPa
Since the calculated tensile stress is less than 0.3fctm,fl, additional bonded reinforcement is not required.

❖ Grouted Tendons
At support (Point A)
Based on Geometry-the same as in unbonded tendons. Hence,
A_{smin} = 1899 mm^2
However, the area of grouted tendon counts toward the requirement
Available reinforcement = 23* 99 = 2277 mm^2
> 1899 mm^2 OK

Based on Computed Stresses
Refer to Fig. 7.4-2 where fcc is the concrete fiber stress in compression; fct is the extreme concrete fiber stress in tension.
f_{ct} = tensile stress = 1.80 MPa
f_{cc} = compressive stress =-5.10 MPa
h-x = 1.80*440/(5.10+1.80) = 115 mm
As = F_t /(5*f_{yk} /8)
Where F_t is the total tensile force over the tensile zone of the entire section.

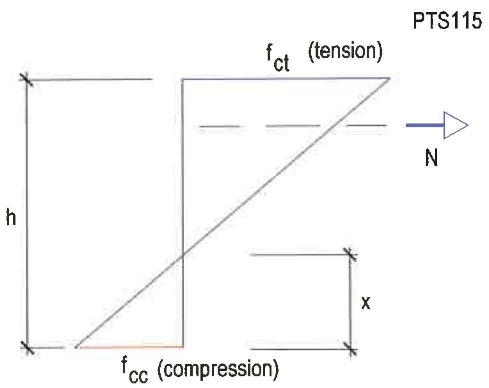


FIGURE 7.4-2 Stress Diagram

F_t = -f_{ct}* b* (h-x)/2 = 1.80*10350*115/(2*1000) = 1071.23 kN
As = 1071.23*1000/(5*460/8) =3726 mm^2
The applicable rebar for this case includes the contribution of bonded tendons. Hence
Aps 23*99 = 2,277 mm^2
As required = 3,726-2,277 = 1,449 mm^2
Comparing (i) and (ii), As = 1,449 mm^2 , since (i) is deemed satisfied
Use 10-16mm bars = 8* 201 mm^2 = 1,608 > 1,449 mm^2 OK

At Span (Point C)
Since the calculated stress is within the threshold value, no rebar needed
The minimum rebar required from different codes is summarized in TABLE 7.4-1

TABLE 7.4-1 Summary of Minimum Rebar (mm^2) (T161SI)

Code	Unbonded		Bonded	
	Support	Span	Support	Span
ACI/IBC	1899	3378	0	0
EC2	0	0	0	0
TR43	1899	0	1449	0

7.5 Deflection Check
Recognizing that (i) the accurate determination of probable deflection is complex [TN292]; and (ii) once a value is determined, the judgment on its adequacy at design time is subjective, and depends on unknown, yet important, parameters such as age of concrete at time of installation of nonstructural members that are likely to be damaged from large displacement, in common construction, deflection checks are generally performed following a simplified procedure. A rigor-

34 TR-43 2nd Edition, Section 5.8.7

ous analysis is initiated, only where the parameters of design and applied loads are more reliably known. In most cases, post-tensioned members are sized according to recommended span/depth ratio proven to perform well in deflection.[35]

The simplified procedure includes:

(i) For visual and functional effects, total long-term deflection from the day supporting shutters are removed not to exceed a value that depending on the code used varies between (span/250 EC2) and (span/240 USA). Camber can be used to offset the impact of displacement.

(ii) Immediate deflection under design live load not to exceed (span/500 for EC2/TR43 designs) or (span/480 for USA).[36]

Both ACI 318/IBC and EC2 (EN 1992-1-1:2004(E)), tie the deflection adequacy to displacement subsequent to the installation of members that are likely to be damaged. This requires knowledge of construction schedule and release of structure for service. In the following the common design practice is followed.

For assessment of long-term displacement in the context of foregoing, ACI 318 recommends a multiplier factor of 2.[37]

Deflections are calculated using a frame analysis program for each of the load cases: dead, live and post-tensioning. Gross cross-sectional area and linear elastic material relationship are used. Point C at the middle of span 4 is selected for deflection check. The values for this point are:

Span 4 Deflection	
Dead Load	5.5 mm
Post-Tensioning	-2.1 mm
Dead Load + PT	3.4 mm
Live Load Deflection	2.1 mm

❖ Long-term Deflection
Multiplier factor assumed for effects of creep and shrinkage on long-term deflection = 2[38]

35 TR-43 5.8.4; ADAPT-TN292

36 Both ACI 318 and EN 1992-1-1:2004(E) tie the deflection check for long-term values subsequent to installation of members that are likely to be damaged from added deflection.

37 ACI 318-11 R9.5.2.5

38 ACI 318 multiplier factor

Load combination for long-term deflection, using a factor of 0.3 for sustained "quasi-permanent" live load:
(1.0*DL + 1.0*PT + 0.3*LL)*(1 + 2)
Long-term deflection: (1 + 2)*(3.4 + 0.3*2.1) = 12.1 mm
Deflection ratio = 12.1/(10.5*1000) = 1/867 < 1/250 OK

❖ Instantaneous Deflection Due to Design Live Load
Live load deflection = 2.1 mm
Deflection ratio = 2.1/(10.5*1000) = 1/5000
<1/480 or 1/500 OK

Deflection does not generally govern the design for members dimensioned within the limits of the recommended values in ACI 318 and balanced within the recommended range, and when subject to loading common in building construction. For such cases, deflections are practically always within the permissible code values.

8 CODE CHECK FOR STRENGTH

8.1 Load Combinations
❖ ACI-318/IBC
1.2*DL + 1.6*LL + 1* Hyp
1.4*DL + 1*HYP

❖ EC2
1.35*DL + 1.5*LL + 1* Hyp

❖ TR43
1.35*DL + 1.5*LL + 0.9* Hyp

For strength combination, the hyperstatic (Hyp) actions (secondary) from prestressing are used. The background for this is explained in detail in reference [Aalami, 1990].

8.2 Determination of Hyperstatic Actions
The hyperstatic moments are calculated from the reactions of the frame analysis under balanced loads from prestressing (Loads shown in Fig. 6.4-4). The reactions obtained from a standard frame analysis are shown in Fig. 8.2-1a. The reactions shown cause the hyperstatic moments in the frame shown in Fig. 8.2-1b.

The hyperstatic (secondary) reactions must be in self-equilibrium, since the applied loading (balanced loads) are in self-equilibrium.

Check the validity of the solution for static equilibrium of the hyperstatic actions using the reactions shown in Fig. 8.2-1a:
ΣVertical Forces = -15.85 + 19.82 +0.42 + 5.529-9.924

= 0.005 ≈ 0 OK
ΣMoments about Support 1 = -82.44*2-17.73*2 +4.87*2 + 26.51*2 + 92.73*2 + 19.82*9 +0.42*19 + 5.529*29.6-9.924*40.1 = -0.05 kNm ≈ 0 OK

8.3 Calculation of Design Moments
The design moment (M_u) is the factored combination of dead, live and hyperstatic moments.

Using ACI/IBC
Design moments are:
M_{U1} = 1.2* M_D + 1.6* M_L + 1.0* M_{HYP}
M_{U2} = 1.4* M_D + 1.0* M_{HYP}

The second combination governs, when the values from dead load are eight times or larger than live loads. This is a rare condition.

By inspection, the second load combination does not govern, and will not be considered in the following.

The factored moments for the codes considered are listed in the following table.

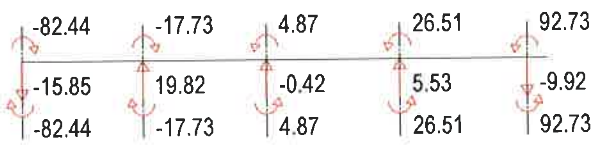
8.4 Strength Design (ULS) for Bending and Ductility
The strength design for bending consists of two provisions, namely

❖ The design capacity (Φ*M_n; R) shall exceed the demand. A combination of prestressing and non-stressed steel provides the design capacity.

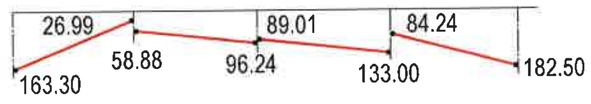
❖ The ductility of the section in bending shall not be less than the limit set in the associated building code. The required ductility is deemed satisfied, if failure of a section in bending is initiated in post-elastic response of its reinforcement, as opposed to crushing of concrete. For the codes covered in this example this is achieved through the limitation imposed on the depth of the compression zone (see Fig. C-8.4-1). The depth of compression zone is generally limited to 50% or less than the distance from the compression fiber to the farthest reinforcement (dr). Since the concrete strain (e_c) at crushing is assumed between 0.003 and 0.0035, the increase in steel strain (e_s) will at minimum be equal to that of concrete at the compression fiber. This will ensure extension of steel beyond its yield point (proof stress) and hence a ductile response.

For expeditious hand calculation, the flexural capacity of a post-tensioned member in common building structures can be approximated by assuming a

PTS458m



(a) Reaction due to balanced loading (kN; kNm)



(b) Hyperstatic moments (kNm)

Post-Tensioning Actions on Design Strip

FIGURE 8.2-1

conservative maximum stress for prestressing tendons. For detailed application of the code-proposed formulas refer to TN179. Application of strain compatibility for the calculation of section capacity is the accurate option (see TN178 for details), but its application for hand calculation is not warranted in daily design work of a consulting office, unless a software is used.

There are two justifications, why the simplified method for ULS design of post-tensioned sections in daily design work are recommended. These are:

(i) Unlike conventionally reinforced concrete, where at each section along a member non-prestressed reinforcement must be provided to resist the design moment, in prestressed members this may not be necessary, since prestressed members possess a

TABLE 8.3-1 Design Moments (T1625I)

Span 4	Point A	Point B	Point C
M_D (kN-m)	-901.00	-319.70	296.89
M_L (kN-m)	-347.40	-126.60	116.20
M_{HYP} (kN-m)	84.24	99.12	133.40
ACI 318-11/IBC 2012 : 1.2DL+1.6LL+1Hyp			
M_{U1} kN-m	-1552.80	-487.08	675.59
EC2 : 1.35DL+1.5LL+1Hyp			
M_{U1} kN-m	-1653.21	-522.38	708.50
TR 43 : 1.35DL+1.5LL+0.9Hyp			
M_{U1} kN-m	-1661.63	-532.29	695.16

base capacity along the entire length of prestressing tendons (Fig. C- 8.4-2a,b). Non-stressed reinforcement is needed at sections, where the moment demand exceeds the base capacity of the section.

(ii) In conventionally reinforced concrete, the stress used for rebar at ULS is a well-defined in the principal building codes. For prestressed sections, however, the stress in tendon at ULS is oftentimes expressed in terms of an involved relationship—hence the tendency to use a simplified, but conservative scheme for everyday hand calculation. For repetitive work, computer programs are recommended.

Using strain compatibility procedure[39] the required reinforcement for each of the three codes are calculated. The outcome is as follows.

❖ **Cracking Moment Larger than Moment Capacity:** Where cracking moment of a section is likely to exceed its design capacity in flexure, reinforcement is added to raise the moment capacity. In such cases, the contribution of each reinforcement is based on the strength it provides. If the minimum value is expressed in terms of cross-sectional area of reinforcement, the applicable value is (A_s + A_{ps}*f_{py}/f_y).

❖ Bonded (Grouted) Tendons

ACI-318[40]/IBC requires that for members reinforced with bonded tendons the total amount of prestressed and nonprestressed shall be adequate to develop a factored load at least 1.2 times the cracking load computed on the basis of the modulus of rupture of the section. In practice, this is taken as cracking moment of the section *Mcr*.

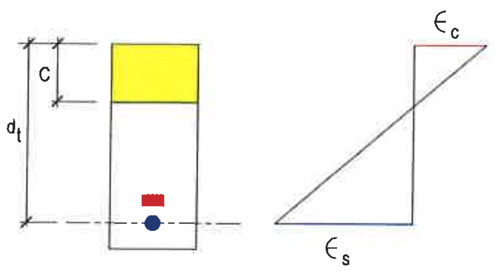
The necessity and amount of rebar is defined as a function of cracking moment of a section (Mcr). For Prestressed Members

Mcr = (fr + P/A)*S
Where, fr is the modulus of rupture defined[41]
Fr = 0.625 √f'c = 0.625 √40 = 3.95 MPa

P/A is the average precompression, and S the section modulus. The Table 7.4-1 summarizes the leading values and the outcome.

[39] ADAPT-TN178
[40] ACI 318-11 Section 18.8.2
[41] ACI 318-11 Section 9.5.2.3

PTS117



(a) Section (b) Strains

Distribution of Strain Over Section

FIGURE C8.4-1 Distribution of Strain over Section

Since at both the face-of-support (section A) and mid-span (section C) the design capacity of the section with prestressing alone exceeds 1.2*Mcr, no additional rebar is required from this provision.

In design situations like above, where the design is initiated by determination of whether a value is less or more than a target, it is advisable to start the check using a simplified, but conservative procedure. If the computed value is close to the target, design check can be followed with a more rigorous computation.

Table 8.4-1 Summary of Required Reinforcement for Strength Limit State (mm²) (T1635I)

Code	Unbonded		Bonded	
	Support	Span	Support	Span
ACI/IBC	3120	1923	1029	0
EC2	4945	2590	3003	726
TR43	4945	2560	3003	726

Assume the following:
Cover to strand CGS = 40 mm;
hence d = h (thickness)-40
Moment arm = 0.9d
Design force in strand = *Aps** 1860 MPa; Φ = 0.9
At face-of-support, with 23 strands, 1860 MPa strength
Φ*Mn = 23* 99*0.9* 1860* (440-40)* 0.9/10⁶ = 1372.21 kNm
The capacity is less than 1.2Mcr=1411 kNm for this section. Rebar has to be added.
Design moment at midspan is calculated in a similar manner.

TABLE 8.4-2 Cracking Moment Values and Parameters (T1645I)

Basic parameters and analysis	Span 4	Section A	Section C
	S_{top} (mm³)	2.45E+08	
	S_{bot} (mm³)		9.93E+07
	P (kN)	2737	2737
	P / A (MPa)	-0.85	-1.1
	fr + (P/A)	4.8	5.05
	Mcr (KNm)	1176	501.46
	1.2 Mcr (kNm)	1411	601
	Φ Mn (kNm)	1372	686
	Status	Added rebar	OK

TABLE 8.4-3 Envelope of Reinforcement for Serviceability (SLS) and Strength Conditions (ULS) (mm²) (T1655I)

Code	Unbonded		Bonded	
	Support	Span	Support	Span
ACI/IBC	3120	3378	1029	0
EC2	4945	2590	3003	726
TR43	4945	4235	3003	726

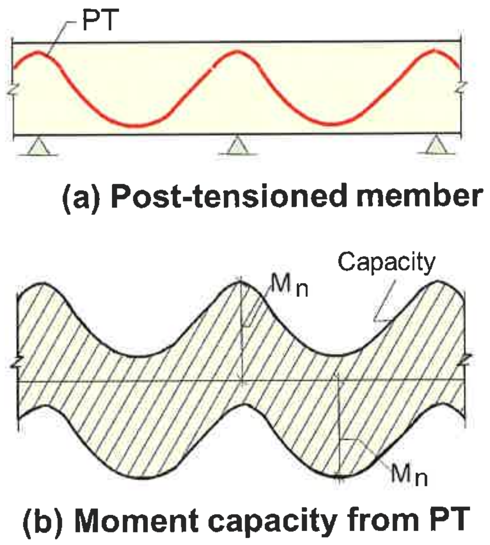
8.5 Punching Shear Check and Design

For moment capacity, the values obtained for a given section using different major building codes do not vary substantially. But, for punching shear, the treatment and outcome differ significantly. Due to the larger variation, the subject matter is treated in greater detail separately (Chapter 4, Section 4.11.6).

9 - CODE CHECK FOR INITIAL CONDITION

At stressing (i) concrete is at low strength; (ii) prestressing force is at its highest value; and (iii) live load generally envisaged to be counteracted by prestressing is absent. As result, the stresses experienced by a member can fall outside the envelope of the limits envisaged for the in-service condition. Hence, post-tensioned members are checked for both tension and compression stresses at transfer of prestressing. Where computed compression stresses exceed the allowable values, stressing is delayed until either concrete gains adequate strength, or the member is loaded. Where computed tension stresses are excessive, ACI/IBC[42] suggest adding non-stressed reinforcement to control cracking.

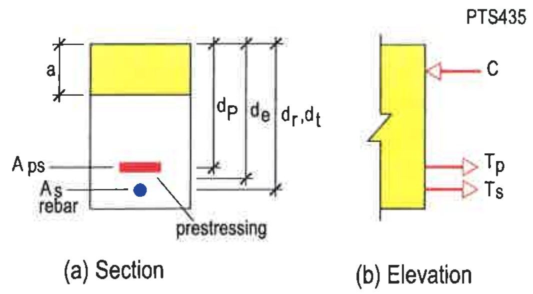
[42] ACI 318-11; Section 18.4



(a) Post-tensioned member

(b) Moment capacity from PT

FIGURE C8.4-2 Demand and Capacity Moments (P500)



Distribution of Basic Forces on a Rectangular Section

FIGURE C8.4-3

9.1 Load Combinations

The codes covered are not specific on the applicable load combination at transfer of prestressing. The following is the combination generally assumed among practicing engineers;

Load Case: 1.0*DL + 0*LL + 1.15*PT

Specification of this design example calls for tendons to be stressed with concrete cylinder reaches 30 MPa. f'_{ci} = 30 MPa[43]

9.2 Stress Check

$\sigma = \pm(M_D + 1.15{*}M_{PT})/S + 1.15{*}P/A$

$S = I/Y_c$

Allowable Stresses

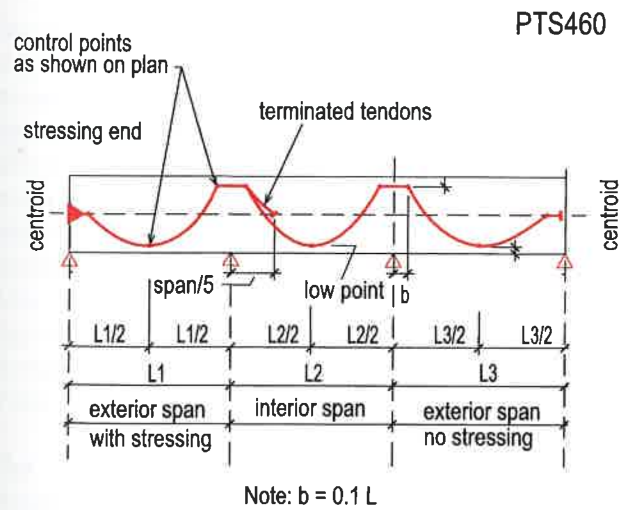
❖ Based on ACI 318-11; IBC 2009

[43] The value specified is on the high side. Most hardware are designed to be stressed at 20MPa concrete cylinder strength or less.

TABLE 9-1 Stresses at Transfer of Post-Tensioning (T1665I)

Span 4	Section A	Section B	Section C
M_D (kN-m)	-901.00	-319.70	296.89
M_{PT} (kN-m)	425.80	234.45	-110.20
P (kN)	2737	2737	2737
P / A (MPa)	-0.85	-1.10	-1.10
f_t (MPa)	0.70	-0.76	-2.98
f_b (MPa)	-3.67	-1.77	0.45
ACI-11/IBC 2009			
F_t (MPa)	1.37	-18	-18
F_b (MPa)	-18	-18	1.37
	OK	OK	OK
EC2			
F_t (MPa)	2.90	-18	-18
F_b (MPa)	-18	-18	2.90
	OK	OK	OK
TR-43			
F_t (MPa)	1.16	-12	-12
F_b (MPa)	-12	-12	1.16
	OK	OK	OK

Note: Section properties I, A, Stop, Sbot are the same as used for service condition stress check F_t and F_b are allowa-ble stresses at top and bottom respectively



Profile for Banded Slab Tendons

FIGURE 10-1

Tension = 0.25* √30 = 1.37 MPa

Compression = 0.60* 30 = -18 MPa

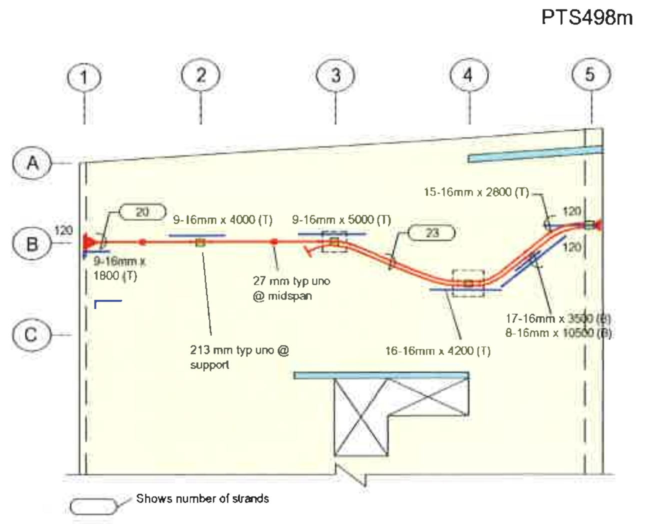


FIGURE 10-2 Layout Using ACI 318 Solution

❖ Based on EC2

Tension = f_{cteff} = 2.90 MPa

Compression = 0.60* 30 = -18 MPa

❖ Based on TR-43

Tension = $0.4f_{ctm}$ = 1.16 MPa

Compression = 0.40* 30 = -12 MPa

Farthest fiber stresses are calculated in a similar manner with to service condition as outlined earlier. The outcome is summarized in the following table.

10 - DETAILING

The final tendon and reinforcement layout for the design strip at line B is shown in figures 10-1 and 10-2 for unbonded tendons. Unbonded tendons are flexible and lend themselves to swerving on plan as shown in the figure. Bonded tendons are not as flexible. They are generally arranged along straight lines.

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CHAPTER 7

POST-TENSIONED BEAM DESIGN STEP-BY-STEP CALCULATION

Post-Tensioned Parking Structure Using Beam Frames and One-Way Slabs (P466)

FOREWORD

The example selected represents a frame of a one-way slab and beam construction—typical of parking structures, or floors, where span in one direction is two or more times the span in the orthogonal direction, for which a beam and one-way slab will be appropriate. The beam frame selected has three spans, each with a different length. The third span is purposely selected to be short, compared to the other two. Also, the optimum post-tensioning for the design is one with different amount of post-tensioning along the length of the structure, and variable profile from span to span.

The objective in selecting a somewhat complex structure is to expose you to the different design scenarios that you generally encounter in real life structures, but are not featured in text books—in particular, where span lengths in a continuous member are widely different.

The example walks you through the 10 steps of design of post-tensioned structures. Aspects of design conditions that are not covered in the design of the example selected, but are important to know, are introduced and discussed as comments or inserted examples.

Design operations that are considered common knowledge, such as the calculation of moments and shears, once the geometry of a structure, its material and loading are known, are not detailed. You are referred to your in-house frame programs.

The design example covers side by side both the unbonded and bonded (grouted) post-tensioning systems, thus providing a direct comparison between the design processes of the two options. In addition, in parallel, the design uses the current American building codes (ACI-318[1] and IBC[2]) along with the European Code (EC2[3]). Where applicable, reference is made to the UK's committee report TR43[4].

[1] ACI 318-11

[2] IBC 12; International Building Code 2012

[3] EN 1992-1-1:2004(E)

[4] TR43-2005; Concrete Society, UK

The common method of analysis for beam frames and one-way slabs is the Simple Frame Method (SFM). While it is practical to use SFM in the environment of consulting firms for design of one-way slabs and beam frames, it becomes laborious if an optimum design for the post-tensioning is sought. The iterative nature of optimization for post-tensioning lends itself well to the application of computer programs, such as ADAPT-PT for expediency in design.

The hand calculations are supplemented by a computer run from ADAPT-PT for verification.

Two text fonts are used in the following. The numerical work that forms part of the actual calculations uses the font shown below:

This font is used for the numerical work of the design.

The following text font is used, wherever comments are made to add clarification to the calculations:

This font is used to add clarification to the calculations.

DESIGN STEPS

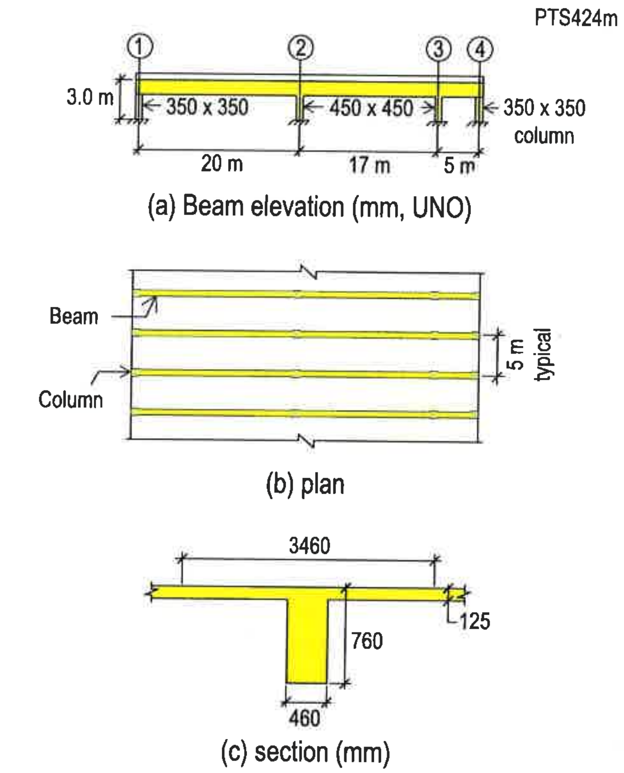
1. GEOMETRY AND STRUCTURAL SYSTEM
   - 1.1 Dimensions and Support Conditions
   - 1.2 Effective Width of Flanges
   - 1.3 Section Properties
2. MATERIAL PROPERTIES
   - 2.1 Concrete
   - 2.2 Nonprestressed Reinforcement
   - 2.3 Prestressing
3. LOADS
   - 3.1 Selfweight
   - 3.2 Superimposed Dead Load
   - 3.3 Live Load
4. DESIGN PARAMETERS
   - 4.1 Applicable Code
   - 4.2 Cover to Rebar and Prestressing Strands
   - 4.3 Allowable Stresses
   - 4.4 Crack Width Limitation
   - 4.5 Allowable Deflection
5. ACTIONS DUE TO DEAD AND LIVE LOADS
6. POST-TENSIONING
   - 6.1 Selection of Design Parameters
   - 6.2 Selection of Post-Tensioning Tendon Force and Profile
   - 6.3 Selection of Number of Strands
   - 6.4 Calculation of Balanced Loads
   - 6.5 Determination of Actions due to Balanced (post-tensioning) Loads
7. CODE CHECK FOR SERVICEABILITY
   - 7.1 Load Combinations
   - 7.2 Stress Check
   - 7.3 Crack Width Control
   - 7.4 Minimum Reinforcement
   - 7.5 Deflection Check
8. CODE CHECK FOR STRENGTH
   - 8.1 Load Combinations
   - 8.2 Determination of Hyperstatic Actions
   - 8.3 Calculation of Design Moments
   - 8.4 Strength Design for Bending and Ductility
   - 8.5 One Way Shear Design
9. CODE CHECK FOR INITIAL CONDITION
   - 9.1 Load Combinations
   - 9.2 Stress Check
10. DETAILING

1 - GEOMETRY AND STRUCTURAL SYSTEM

The floor consists of a one-way slab supported on parallel beams as shown in Fig. 1-1.

1.1 Dimensions and Support Conditions

❖ Geometry is as shown in Fig. 1-1(a) and (b)
❖ Beam cross section as shown in Fig. 1-1(c)
❖ Total tributary width = 5 m typical
❖ Columns extend below the deck only; first and last columns are assumed hinged at the bottom.



Beam Frame Geometry

FIGURE 1-1

TABLE 1.3-1 Section Properties (T131SI)

	Spans 1 and 2		Span 3	
	Axial effects	Bending effects	Axial effects	Bending effects
Area (mm^2)	9.171e+5	5.996e+5	9.171e+5	4.484e+5
I in.^{4} (mm^4)	-----	3.185e+10	-----	2.472e+10
Y_t in. (mm)	184	248	184	310
Y_b in. (mm)	576	512	576	450
S_{top} (mm^3)	-----	1.28e+8	------	7.97e+7
S_{bot} (mm^3)	------	6.22e+7	-------	5.49e+7

I = Second moment of area (moment of inertia);
Y_t = distance of centroid to top fiber of section;
Y_b = distance of centroid to bottom fiber of section;
S_{top} = section modulus for top fiber; (I/Y_t); and
S_{bot} = section modulus for bottom fiber; (I/Y_{bot}).

End columns are assumed hinged and detailed as hinged at the connection to the footing, in order to reduce stresses and potential of cracking due to shrinkage and creep of concrete for the first elevated deck.

1.2 Effective Width of Flanges

When hand calculation is used in analysis of flanged beams, an effective width is selected to account for the bending effects of the structure. ACI-318-11[5] explicitly states that the effective width used for analysis of conventionally reinforced flanged beams does not apply when the same is post-tensioned, but does not clarify the alternative. Section 4.8.3 outlines the reason behind ACI-318's standing and explains the applicable procedure. Briefly, for axial forces (post-tensioning) the entire cross-sectional area is effective. But, for computation of flexural stresses in hand calculation a reduced flange width is applicable.

❖ For axial effects (precompression) use the entire tributary of the structure
❖ For bending effects use the "effective width" value associated with the bending of the flanged beam.

Also, note that the effective width concept is associated with the distribution of elastic stresses in the flange of a beam. It is applicable for "serviceability limit" design (SLS) of a post-tensioned member. For safety checks (ULS) the effective width does not apply.

Other codes and TR43 covered herein are also mute on the effective width of a post-tensioned flanged beam. For conventionally reinforced concrete, ACI 318-11[6] recommends the least of the following values for effective width of an interior span in bending:

(i) eight times the flange thickness on each side of the stem,
(ii) one quarter of the span, or
(iii) the beam's tributary.

Tributary width = 5000 mm
(i) Sixteen times flange thickness plus stem width
= 16*125 + 460 = 2460 mm
(ii) One quarter of span
For span 1 = (20*1000)/4 = 5000mm
For span 2 = (17*1000)/4 = 4250mm
For span 3 = (5*1000)/4 = 1250mm
(iii) Tributary width = 5000 mm
Assume the following:
Spans 1 and 2: 2460 mm
Span 3: 1250 mm

1.3 Section Properties

The section properties for the axial effects are the same for all spans. For bending effects, however, due to different effective widths, the section properties differ. The section properties calculated are listed in Table 1.3-1.

I = Second moment of area (moment of inertia);
Y_t = distance of centroid to top fiber of section;
Y_b = distance of centroid to bottom fiber of section;
S_{top} = section modulus for top fiber; (I/Y_t); and
S_{bot} = section modulus for bottom fiber; (I/Y_{bot}).

2 - MATERIAL PROPERTIES

2.1 Concrete

Cylinder strength f'_c, f_{ck} (28 day) = 28 MPa

[5] ACI 318-11, Section 18.1.3
[6] ACI 318-11, Section 8.12.2

Weight = 24 kN/m^3
Modulus of Elasticity = 4700 $\sqrt{f'c}$ = 24870 MPa [ACI]
= 22* 10^3* [(f_{ck} +8)/ 10]$^{0.3}$ [EC2, TR-43];
= 32308 MPa
Creep coefficient t = 2
Material factor, γ_c = 1 – ACI, 1.50 [EC2, TR-43]
Strength at transfer, f_{ci} = 20 MPa

The creep coefficient is used to estimate the long-term deflection of the slab.

2.2 Nonprestressed (Passive) Reinforcement

f_y = 460 MPa
Elastic Modulus = 200000 MPa
Material factor, γ_c = 1 [ACI]; 1.15 – [EC2, TR-43]
Strength reduction factor (bending), φ = 0.9 [ACI];
= 1 [EC2, TR-43]

2.3 Prestressing: (Figs 2.3-1 through 2.3-4)

Material—low relaxation, seven wire ASTM 416 strand
Nominal strand diameter = 13 mm
Strand area = 99 mm^2
Elastic Modulus = 200000 MPa
Ultimate strength of strand (f_{pu}) = 1860 MPa
Material factor, γ_c = 1 [ACI]; 1.15 [EC2, TR-43]

System

Unbonded System

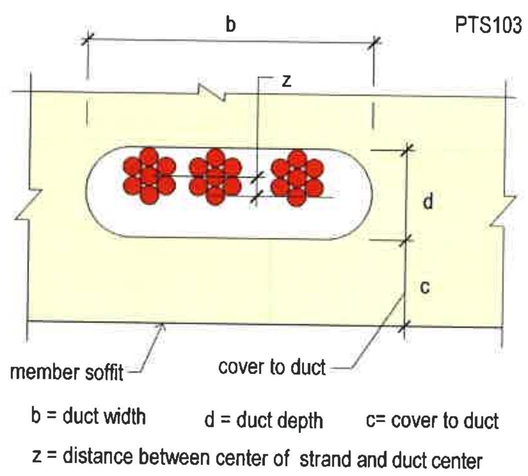
Angular coefficient of friction (μ) = 0.07
Wobble coefficient of friction (K) = 0.003 rad/m
Anchor set (wedge draw-in) = 6 mm
Stressing force = 80% of specified ultimate strength
Effective stress after all losses[8] = 1200 MPa

Bonded System

Use flat ducts 20x80mm; 0.35 mm thick metal sheet housing up to five strands
Angular coefficient of Friction (μ) = 0.2
Wobble coefficient of Friction (K) = 0.003 rad/m
Anchor set (Wedge Draw-in) = 6 mm
Offset of strand to duct centroid (z) = 3 mm
Effective stress after all losses = 1100 MPa

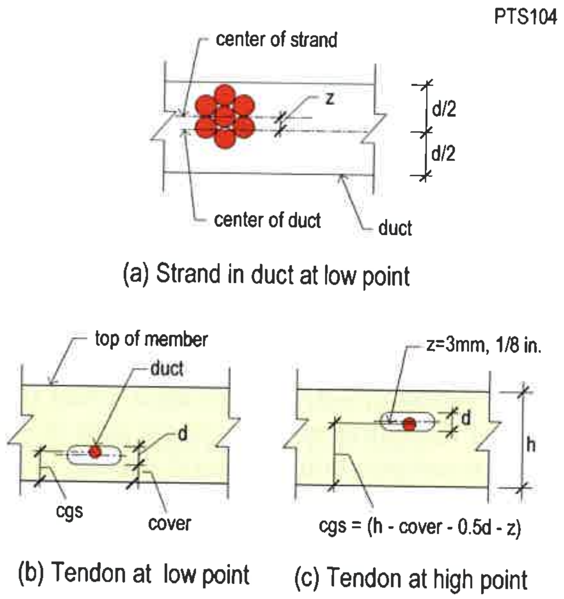
Section through the bonded tendon duct in place is shown in Fig. 2.3-1 and 2.3-2

[7] EN 1992-1-1:2004(E) Table 3.1
[8] For hand calculation, an effective stress of tendon is used. The effective stress is the average stress along the length of a tendon after all immediate and long-term losses. The value selected for effective stresses is a conservative estimate. When "effective stress" is used in design, the stressed lengths of tendons are kept short, as it is described later in the calculations.



Section through a Flat Duct at Low Point

FIGURE 2.3-1 Bonded Tendon Section



Position of Center of Gravity (cgs) of Strand at Extreme Positions in Member

FIGURE 2.3-2

3 - LOADS

3.1 Selfweight

Slab = 0.125 m*2400 kg/m^3*5m*9.806/1000
= 14.71 kN/m
Stem = 0.635*0.460*2400*9.806/1000
= 6.87 kN/m
Total selfweight = 14.71 + 6.87 = 21.58 kN/m

3.2 Superimposed Dead Load

From mechanical, sealant and overlay 0.5 kN/m^2
= 0.5 kN/m^2*5 m= 2.5 kN/m
Total Dead Load = 21.58 + 2.5 = 24.08 kN/m^2

3.3 Live Load:[9] 2.5 kN/m^2

Total live load = 2.5*5 = 12.5 kN/m
MaxLL/DL ratio = 12.5/24.08 = 0.52 < 0.75 ∴ Do not skip live loading

Strictly speaking, live loads must be skipped (patterned) to maximize the design values. But, when the ratio of live to dead load is small (less than 0.75), it is adequate to determine the design actions based on full value of live loads on all spans (ACI-318-11[10]). This is specified for slab construction, but it is also used for beams.

4 - DESIGN PARAMETERS

4.1 Applicable Codes

The design is carried out according to each of the following codes. Further, reference is made to the Committee Report TR43, where appropriate.

- ACI 318-2011; IBC-2012
- EC2 (EN 1992-1-1:2004)

4.2 Cover to rebar and prestressing strands

Unbonded and bonded system
Minimum rebar cover = 40 mm top and bottom

The cover selected is higher than the minimum code requirement to allow for installation of top slab bars over the beam cage in the transverse direction to the beam.

Minimum prestressing CGS =70 mm

The cover and hence distance to the CGS (Center of Gravity of Strand) is determined by the requirements for fire resistivity and positioning of tendons within the beam cage. The distance 70 mm selected is slightly higher than the minimum required. Its selection is based on ease of placement.

[9] The live load assumed is somewhat high. The common value in the US, based on ASCE 07 is 2 kN/m2. Also, US codes allow reduction of live load under certain conditions. In this example, the higher value common in a number of world regions is used and the value is not reduced.
[10] ACI 318-11, Section 13.7.6

4.3 Allowable Stresses

A. Based on ACI 318-11/IBC 2012[11]

Allowable stresses in concrete are the same for bonded and unbonded PT systems

- For sustained load condition

Compression = 0.45*f'_c = 12.60 MPa

- For total load condition

Compression = 0.60*f'_c = 16.80 MPa

Tension: (Transition condition of design is targeted)
The range for transition (moderate cracking) is as follows:
= 0.62*$\sqrt{f'_c}$ < stress ≤ 1.00*$\sqrt{f'_c}$
= 3.28 MPa < stress ≤ 5.29 MPa

For top fibers the lower value will be targeted, in order to limit crack width and improve durability. For the bottom fiber the higher value will be used, allowing for a wider crack width

- For initial condition

Compression = 0.60 f'_{ci} = 0.6* 20 = 12 MPa
Tension = 0.25 $\sqrt{f'_c}$ = 1.12 MPa

For one-way systems, ACI 318-11 defines three conditions of design, namely uncracked (*U*), transition (*T*) and cracked (*C*). The three conditions are distinguished by the magnitude of the maximum hypothetical tension stress in concrete at the farthest tension fiber. For the current design example the transition (*T*) condition is selected. For this condition, hypothetical tension stresses can exceed 0.62$\sqrt{f'_c}$ but not larger than 1.00$\sqrt{f'_c}$ However, since the surface of the parking structure being designed is exposed, the design example uses a stress limit of 0.75$\sqrt{f'_c}$ for the top surface and the maximum value allowed by the code for the bottom surface. This is not a code requirement. Based on code, 1.00$\sqrt{f'_c}$ would have been acceptable. The selection of a lower value for the top surface is based on good engineering practice.

B. Based on EC2[12]

EC2 does not specify "limiting" allowable stresses in the strict sense of the word. There are stress thresholds that trigger crack control. These are the same for both bonded and unbonded systems. For computed stressed below the code thresholds, the minimum reinforcement requirement provisions of EC2 suffices.

- For "frequent" load condition

[11] ACI 318-11, Sections 18.3 and 18.4
[12] EN 1992-1-1:2004(E), section 7.2

Concrete:
Compression = $0.60*f_{ck} = 0.6*28 = 16.80$ MPa
Tension (concrete) $F_t = f_{ct,eff} = f_{ctm}$[13]
$F_t = 0.30*f_{ck}{}^{(2/3)} = 0.30*28^{(2/3)}$
$= 2.77$ MPa (Table 3.1, EC2)
Tension (nonstressed steel) $= 0.80*f_{yk} = 0.8*460$
$= 368$ MPa
Tension (prestressing steel) $= 0.75*f_{pk}$
$= 0.75*1860 = 1395$ MPa

❖ For "quasi-permanent" load condition
Compression $= 0.45*f_{ck} = 0.45*28 = 12.60$ MPa
Tension (concrete) = 2.77 MPa
same as frequent load combination

Unlike ACI-318/IBC, provisions in EC2 permit[14] overriding the allowable hypothetical tension stress in concrete, provided cracking is controlled not to exceed the allowable values.

❖ For "initial" load condition (Table 3.1; EC2)
Tension (Unbonded) $= f_{ct,eff} = f_{ctm}$
$0.30*f_{ci}{}^{(2/3)} = 0.30*20^{(2/3)} = 2.21$ Mpa
Compression[15] $= 0.60*f_{ci} = 0.6*20 = 12$ MPa

C. Based on TR-43[16]
Unbonded tendons
For "frequent" load combination
Tension $= 1.35\ f_{ctm,fl}$
$f_{ctm,fl}$ = larger of (1.6- h/1000) f_{ctm} or f_{ctm}[17]
= larger of (1.6- 0.760) f_{ctm} or f_{ctm}
= larger of $0.84*f_{ctm}$ or f_{ctm}
$f_{ctm} = 0.30*f_{ck}{}^{(2/3)}$ (Table 3.1, EC2)
$= 0.30*28^{(2/3)} = 2.77$ MPa
Allowable tension stress $= 1.35*2.77 = 3.74$ MPa

Bonded tendons
For "frequent" load combination
For the members with 0.2 mm allowable crack width, allowable tension stress without bonded reinforcement is:
Tension $= 1.65\ f_{ctm,fl} = 1.65*2.77 = 4.57$ MPa
For tension (with bonded reinforcement)
Tension $= 0.3f_{ck} = 0.30*28 = 8.40$ MPa
Compression $= 0.6*f_{ck} = 0.6*28 = 16.80$ Mpa
TR-43 specifies allowable concrete compressive stresses for bonded PT systems, but is mute for unbonded systems. In practice, the same values are used for both systems.

For "quasi-permanent" load combination:
Allowable tension stresses are the same as "frequent" load condition.
Compression $= 0.45*f_{ck} = 0.45*28 = 12.60$ MPa

❖ For "initial" load condition[18]
Tension $= 0.72\ f_{ctm}$
$f_{ctm} = 0.30*f_{ci}{}^{(2/3)}$ (Table 3.1, EC2)
$= 0.30*20^{(2/3)} = 2.21$ MPa
Allowable tension stress $= 0.72*2.21 = 1.59$ MPa
Compression $= 0.50*f_{ci} = -10$ MPa

4.4 Crack Width Limitation
A. Based on ACI 318-11/IBC 2012
Crack width control and limitation applies when member is designed for the "cracked" regime. No requirements are stipulated, if as in this example, the stresses are kept within the uncracked (U) or transition (T) regime.

B. Based on EC2[19]
In EC2, the allowable crack width depends on whether the post-tensioning system used is "bonded," or "unbonded," and the load combination being considered. Frequent load condition:

❖ Prestressed members with bonded tendons
0.2 mm; to be checked for frequent load case
❖ Prestressed members with unbonded tendons
0.3 mm; to be checked at quasi-permanent load case

C. Based on TR-43[20]
For all members = 0.2 mm

4.5 Allowable Deflection
A. Based on ACI 318-11/IBC 2012[21]
In all major codes, the allowable deflection is tied to (i) the impact of the vertical displacement on occupants; (ii) the possible damage to installed nonstructural objects such as partitions, glass, or floor covering; and (iii) functional impairment, such as proper drainage. Details of the allowable values, their measurement and evaluation are given in Chapter 4. For perception of displacement by sensitive persons, consensus is limit of L/240, where L is the deflection span. It is important to note that this is the displacement that can be observed by a viewer.

13 EN 1992-1-1:2004(E) , section 7.3.2(4)
14 EN 1992-1-1:2004(E) , section 7.3.2(4)
15 EN 1992-1-1:2004(E) , section 5.10.2.2(5)
16 TR-43 Second Edition, Table 3. For tensile stress, stress limit without bonded reinforcement is considered.
17 EN 1992-1-1:2004(E) , Eqn.3-23
18 TR-43 Second Edition, Section 5.8.2.
19 EN 1992-1-1:2004(E), Table 7.1N
20 TR-43 Second Edition, Section 5.8.3.
21 ACI 318-11, Section 18.3.5

❖ Since in this design example there is no topping on the finished slab, the applicable vertical displacement is the total deflection subsequent to the removal of forms.
❖ The deflection check for potential damage to nonstructural elements is not applicable in this case, since the structure is a frame of an open parking structure.
❖ The drainage and ponding of water will be controlled through proper sloping of the floors.
Total allowable deflection: L/240
The frame will be provided with a camber to minimize the impact of deflection.

B. Based on EC2[22]
The interpretation and the magnitude of allowable deflections in EC2 are essentially the same as that of ACI-318. The impact of vertical displacement on the function of the installed members and the visual impact on occupants determine the allowable values. The following are suggested values:

Deflection subsequent to finishing of floors from quasi-permanent combination: L/250
The frame will be provided with a camber to minimize the impact of deflection.

C. Based on TR-43[23]
TR43 refers to EC2 for allowable deflections.

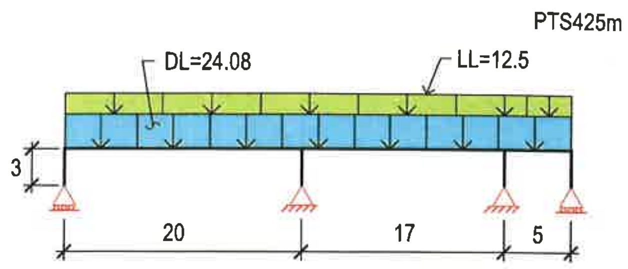
In summary, the allowable deflection from the two codes and the committee report are essentially the same. Conservatively, it can be summarized as follows:

Total deflection from quasi-permanent load combination - L/250
Where, L is the length of the span.

5. ACTIONS DUE TO DEAD AND LIVE LOADING

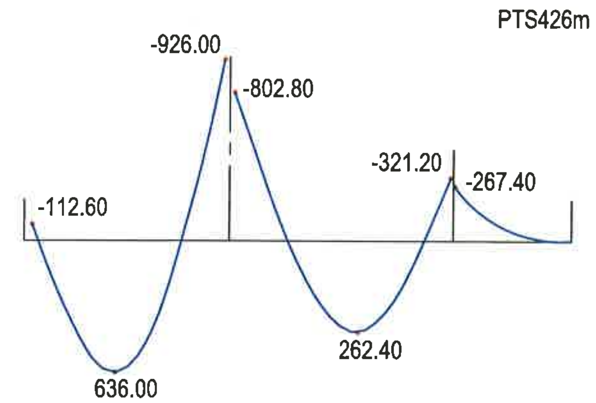
The structural system of the frame and its dead and live loading are shown in Figs. 5-1 through 5-3.

Actions due to dead and live loads are calculated for this example using a generic frame analysis program. The members are assumed prismatic and of uniform cross section throughout the length of each span. Spans 1 and 2 have the same geometry. Centerline to centerline distances are used for span lengths. No allowance is made in the hand calculation for stiffening of members over support. Some software accounts for



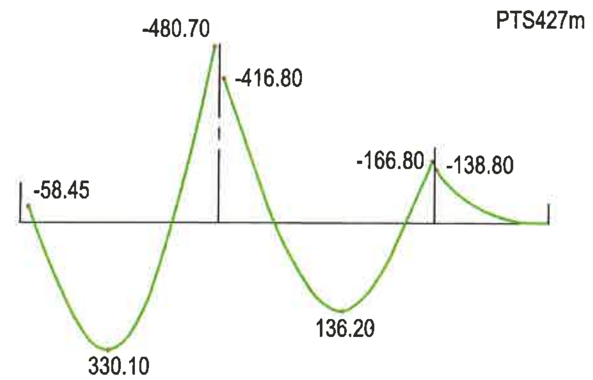
Structural Frame and its Dead and Live Loading (kNm;m)

FIGURE 5-1



Dead Load Moment Distribution (kN-m)

FIGURE 5-2



Live Load Moment Distribution (kNm)

FIGURE 5-3

22 EN 1992-1-1:2004(E), Section 7.4.1
23 TR-43 Second Edition, Section 5.8.4.

TABLE 5-1 Moments at Face-of-Supports and Midspans (T132)

	Span 1			Span 2			Span 3		
	Left	Mid	Right	Left	Mid	Right	Left	Mid	Right
M_D (kN-m)	-112.60	636.00	-926.00	-802.80	262.40	-321.20	-267.40	-68.88	5.27
M_L (kN-m)	-58.45	330.10	-480.70	-416.80	136.20	-166.80	-138.80	-35.75	2.73
$M_D + 0.3M_L$ Sustained load (kN-m))	-130.14	735.03	-1070.21	-927.84	303.26	-371.24	-309.04	-79.61	6.09
$M_D + M_L$ Total load (kN-m)	-171.05	966.1	-1406.7	-1219.6	398.6	-488	-406.2	-104.63	8

this stiffening and increase the moment of inertia of the beam over the support region [ADAPT-PT, 2012]. The centerline moments calculated are reduced to the face-of-support using the static equilibrium of each span.

The critical design moments are not generally at midspan. But, for hand calculation, the midspan location is selected. The approximation is acceptable when spans and loads are essentially uniform.

The computed moments from the frame analysis are reduced to the face of each support using statics of respective span. The face-of-support moments and the moments at midspan are summarized in Table 5-1.

6. POST-TENSIONING

6.1 Selection of Design Parameters

Unlike conventionally reinforced members, where given geometry, boundary conditions, material properties and loads result in a unique design, for post-tensioned members in addition to the above a minimum of two other input assumptions are required, before a design can be concluded. A common practice is (i) to assume a level of precompression and (ii) target to balance a percentage of the structure's dead load. In this example, based on experience the level of precompression suggested is larger than the minimum required by ACI-318 code (0.86 MPa). Other major building codes do not specify a minimum precompression. Rather, they specify a minimum reinforcement. Use the following assumption to initiate the calculations.

Minimum average precompression = 1.0 MPa
Maximum average precompression = 2.0 MPa
Target balanced loading =60 % of total dead load

Based on experience for economy of design, a minimum precompression of 1.0 MPa over the entire section is assumed. ACI 318 stipulates a minimum of 0.86 MPa. In the manual calculation, the minimum precompression is used as an entry value (first trial) for design. The stipulation for a maximum precompression does not enter the hand calculation directly. It is stated as a guide for a not-to-exceed upper value. For deflection control the selfweight of the critical span is recommended to be balanced to a minimum of 60% of its selfweight [Aalami, et al, 2003]. Other spans need not be balanced to the same extent. As it will become apparent further in the calculations, for the current beam frame it is beneficial if the tendon exerts a downward force on the third span, as opposed to an upward force in the critical (first) span.

Effective stress in prestressing strand:
For unbonded tendons: f_{se} = 1200 MPa
For bonded tendons: f_{se} = 1100 MPa

The design of a post-tensioned member can be based either on the "effective force", or the "tendon selection" procedure. In the effective force procedure, the average stress in a tendon after all losses is used in design. In this case, the design concludes with the total effective post-tensioning force required at each location. The total force arrived at the conclusion of design is then used to determine the number of strands required, with due allowance for friction and long-term losses. This provides an expeditious and

simple design procedure for hand calculations. In the "tendon selection" procedure, the design is based on the number of strands with due allowance for the immediate and long-term losses. In the following, the "effective force" method is used to initiate the design. Once the design force is determined, it is converted to the number of strands required.

The effective stress assumed in a strand is based on the statistical analysis of common floor slab dimensions for the following conditions (Fig. C6.1-1):
(i) Members have dimensions common in building construction;
(ii) Tendons equal or less than 38 m long stressed at one end. Tendons longer than 38m, but not exceeding 76m are stressed at both ends. Tendons longer than 76m are stressed at intermediate points to limit the unstressed lengths to 38m for one-end stressing or 76m for two-end stressing, whichever be applicable;
(iii) Strands used are the commonly available 13 or 15 mm nominal diameter with industry common friction coefficients as stated in material properties section of this design example; and
(iv) Tendons are stressed to 0.8fpu.

For other conditions, a lower effective stress is assumed, or tendons are stressed at intermediate points. In the current design, the total length of the tendon is 41 m. It is stressed at both ends. Detailed stress loss calculations, not included herein, indicate that the effective tendon stress is 1250 MPa for the unbonded system and also larger than assumed for the grouted system.

6.2 Selection of Post-Tensioning Tendon Force and Profile

The design prestressing force in each span will be chosen to match a whole number of prestressing strands. The following values are used:

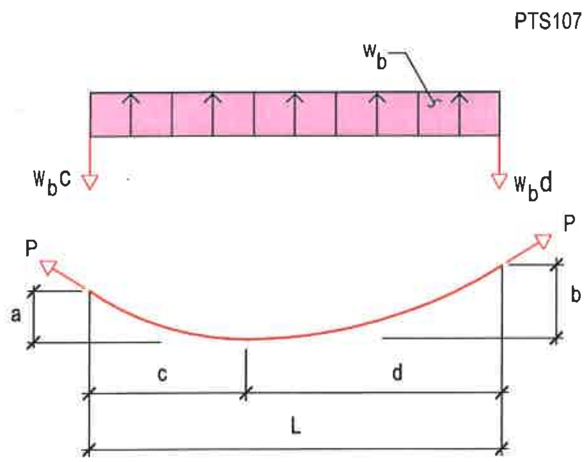
1. The effective force along the length of each tendon is assumed to be constant. It is the average of force distribution along a tendon.

Unbonded tendons
Force per tendon = 1200*99 mm²/1000
= 118.8 ≈ 119.0 kN/ tendon
Use multiples of 119 kN when selecting the post-tensioning forces for design.

Bonded tendons
Force per tendons = 1100*99 mm²/1000
= 108.9 ≈ 109.0 kN/ tendon
Use multiples of 109 kN when selecting the post-tensioning forces for design.

2. Tendon profiles are chosen to be simple parabola. These produce a uniform upward force in each span.

For ease of calculation the tendon profile in each span is chosen to be concave upward, simple parabola from centerline to centerline of supports (Fig. C6.2-1). The position of the low point is selected such as to generate a uniform upward force in each span. The relationship given in Fig. C6.2-1 defines the profile. For exterior spans, where the tendon high points are not generally the same, the resulting low point will not be at midspan. For interior spans, where tendon high points are the same, the low point will coincide with midspan. Obviously, the chosen profile is an approximation of the actual tendon profile used in construction. Sharp changes in curvature associated with the simple parabola profile assumed are impractical to achieve on site. The tendon profile at construction is likely to be closer to reversed parabola, for which the distribution of lateral tendon forces will be somewhat different as discussed henceforth. Tendon profiles in construction and the associated tendon forces are closer to the diagrams shown in Fig. C6.2-2.



$c/L = [\sqrt{a/b} \,/\, (1 + \sqrt{a/b})]$

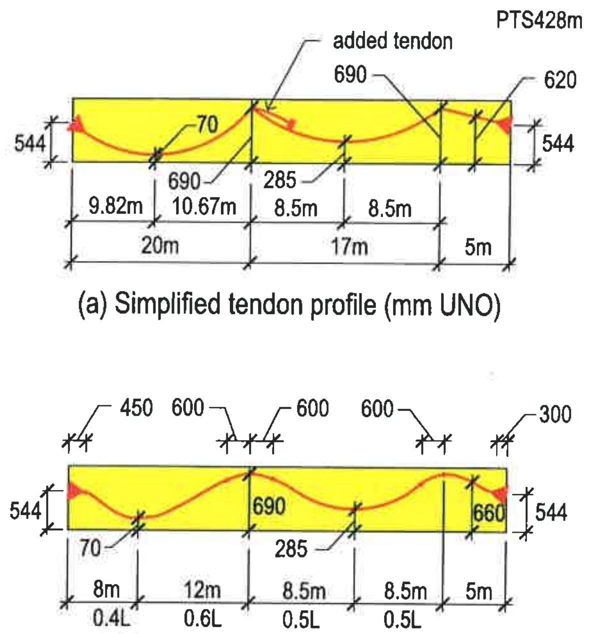
$w_b = 2aP/c^2$

Geometry and Actions of a Parabolic Tendon

FIGURE 6.2-1

6.3 Selection of Number of Strands

Determine the initial selection of number of strands for each span based on the assumed average precompres-



(a) Simplified tendon profile (mm UNO)

(b) Actual tendon profile used in construction (mm UNO)

Comparison of Simplified and Actual Tendon Profiles

FIGURE 6.2-2

sion and the associated cross-sectional area of each span's tributary. Then, adjust the number of strands selected, based on the uplift they provide.

Unbonded tendons

1.0N/mm²*9.171e+5 mm²/1000 = 917.1 kN
Number of strands = 917.1 kN/119 kN = 7.71
9 strands selected (8 strands would work too)
Force in 9 strands = 9*119 = 1071 kN

Bonded tendons

Number of strands = 917.1 kN/109 kN = 8.4
9 strands selected.

It is noted that the number of strands required to satisfy the same criterion differs between the unbonded and bonded systems. Due to higher friction losses, when using the bonded system, generally more strands are needed to satisfy the in-service condition of design. For brevity, without compromising the process of calculation, in the following the same number of strands is selected for both systems.

6.4 Calculation of Balanced Loads

Balanced loads are the forces that a tendon exerts to its concrete container. It is generally broken down to forces normal to the centerline of the member (causing bending) and axial to it (causing uniform precompression) and added moments at locations of change in centroidal axis. Fig. C6.2-2 shows two examples of balanced loading for members of uniform thickness.

Span 1

The profile of the first span is chosen such that the upward force on the structure due to the tendon is uniform. This is done by choosing the location of the low point so that in each span the profile is a continuous simple parabola (Fig. C6.2-1). Span 1 is the longest span and it is considered the critical span. It is designed with maximum drape, in order to utilize the maximum amount of balanced loading in the most critical span. If the low point of the tendon is not selected at the location determined by "c", two distinct parabolas result. The upward force from a single parabolic profile selected is shown in Fig.C 6.2-1.

Refer to Fig. C6.2-1 and C6.2-2
a = 576 – 70 = 506 mm
b = 690 – 70 = 620 mm
L = 20.0 m
c = [506/620]$^{0.5}$/[1 + (506/620)$^{0.5}$]}*20.0
= 9.49 m
W_b = 1071 kN*2*0.506/9.49² = 1071*0.01124/m
= 12.04 kN/m
% DL balanced = 12.04/24.08 = 50% < 60% NG
Prorated number of strands = (60%/50%)*9
=10.8 strands; use 12 strands
Force = 12*119 kN =1428 kN
W_b = 1428*0.01124/m = 16.05 kN/m ↑
% DL balanced = 16.05/24.08 = 67% OK
Balanced load reaction, left = 16.05 kN/m*9.49
= 152.31 kN↓
Balanced load reaction, right= 16.05 kN/m*10.51
= 168.69kN↓
Fig. 6.4-1 shows the distribution of balanced loading for span 1.

Span 2
Continuous Tendons

This span is shorter and not the critical span. Therefore, the 9 tendons necessary for the assumed minimum precompression of 1.0 MPa is used. In addition, recognizing that balancing a lower percentage of selfweight will be beneficial to the critical span (span 1); the minimum 60% used as guideline is waived for this span. A smaller percentage for balanced loading is preferred. The dead load in span 2 reduces the design moment of span 1. Balancing more dead load in span 2 will not be beneficial to the design. Also note that the tendon low point is located at midspan.

W_b = 50%*24.08 kN/m =12.04 kN/m ↑
a = W_b*L²/8*P = [(12.04*17²)/(8*1071)]*1000
= 405 mm
CGS =690 – 405 = 285 mm
Balanced load reactions = 12.04 kN/m*8.5 m
=102.34 kN ↓ (Left and Right)

Added Tendons

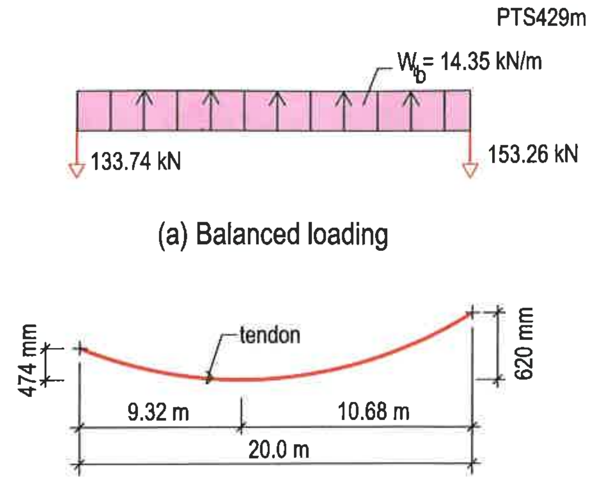
Reduction of tendons from 12 in span 1 to 9 in span 2 means that 3 tendons from span 1 terminate in span 2. The terminated three tendons are dead-ended in span 2. The dead-end is located at a distance *0.20L* from the left support at the centroid of the beam section. The tails of the terminated tendons are assumed to be in the shape of a half parabola with its apex horizontal over the support and concave downward to the dead end. Hence, the vertical balanced loading of these tendons will be downward, with a concentrated upward force at the dead end (Fig. 6.4-2). The magnitude of the downward vertical force W_b is:

W_b = (2aP)/c²
a = 690 – 576 = 114 mm
c = 0.20*17 = 3.4 m
W_b = (2aP)/c² = (2*0.114 *3*119)/3.4²)
= 3*119*0.0197 = 7.04 kN/m ↓
Concentrated force at dead end = 7.04*3.4 m
= 23.94 kN ↑

Span 3

The tendon profile in this span is chosen to be straight from the high point at the interior support, to the centroid of the section at the exterior support. The objective is to avoid uplift in the short span. As a matter of fact, for this beam a downward force in the third span would be beneficial to the design of the interior span, since in this span the distribution of dead load moment is all negative.

CGS left = 690 mm
CGS right = 576 mm
CGS center = (690 + 576)/2 = 633 mm
Vertical balanced loading forces are concentrated forces acting at the supports only, they are equal and opposite. Force is calculated using the tangent of the tendon slope for the small angle.
W_b = 1071 kN*(690 – 576)/(5*1000)
= 24.42 kN ↑(right); ↓ (left)

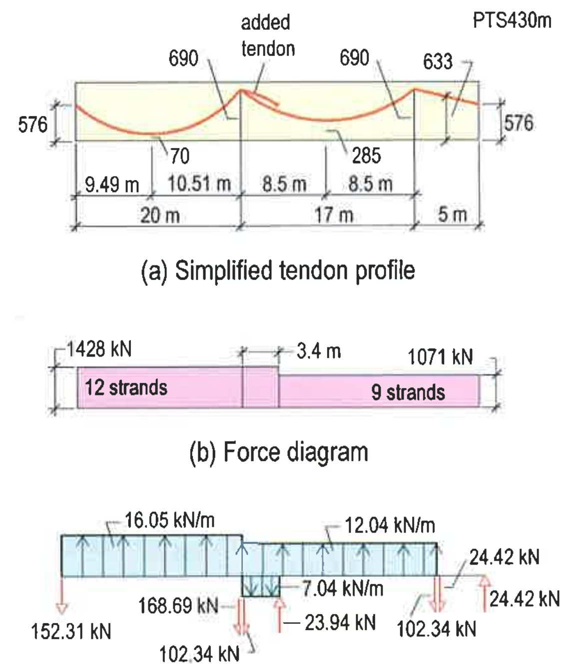


(a) Balanced loading

(b) Simple parabola

Tendon and Balanced Loading for Span 1

FIGURE 6.4-1



(a) Simplified tendon profile

(b) Force diagram

(c) Balanced loading

Tendon, Force and Balanced Loading

FIGURE 6.4-2

The complete tendon profile, effective force and balanced loading diagram is shown in Fig. 6.4-2.

Verify the computed balanced loading

(i) Sum of vertical forces must add up to zero:

-152.31 - 168.69 + 16.05 *20 - 102.34 + 23.94 - 7.04 *3.4 + 12.04*17 – 102.34 - 24.42 + 24.42
= 0.004 OK

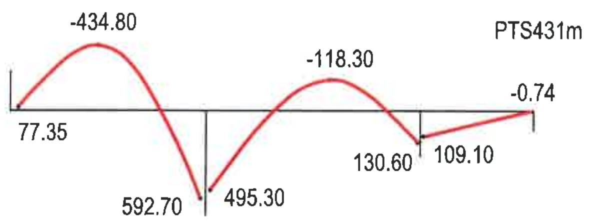
(ii) Sum of moments of the forces must be zero. Taking moments about the first support gives:

-168.69*20 + 16.05 *20²/2 – 102.34*20 –7.04 *3.4*(20+3.4/2) + 23.94 *23.4 + 12.04*17*(20+17/2) – 102.34*37 – 24.42*37 + 24.42*42 = 0.92 kN-m OK

The forces exerted by a tendon to its container (beam frame in this case) are always in static equilibrium, regardless of the geometry of tendon and the configuration of the member that contains the tendon. To guarantee a correct solution, it is critical to perform an equilibrium check for the balanced loads calculated (Fig. C6.4-2) before proceeding to the next step. And, that the concentrated forces over the supports are correctly computed and accounted for. In particular, the force due to the short length of the terminated strands in the second span must be included to satisfy equilibrium. If equilibrium is not satisfied, it becomes imperative to ensure that the results err on the conservative side.

6.5 Determination of Actions due to Balanced (Post-Tensioning) Loads

The distributions of post-tensioning moments due to balanced loading are shown in Fig. 6.5-1. These actions are obtained by applying the balanced loads shown in Fig. 6.4-2(c) to the frame shown in Fig. 5-1. The moments shown in the figure are those reduced to the face-of-support. Midspan moments are also shown in the figure.



Post-Tensioning Moment Distribution (kNm)

FIGURE 6.5-1

Actions due to post-tensioning are calculated using a standard frame program. The input geometry and boundary conditions to the standard frame program are the same as used for the dead and live loads.

7 - CODE CHECK FOR SERVICEABILITY

7.1 Load Combinations

The following lists the recommended load combinations of the building codes covered for serviceability limit state (SLS).

❖ [ACI, IBC]
Total load condition: 1*DL + 1*LL + 1*PT
Sustained load condition: 1*DL + 0.3*LL + 1*PT[24]

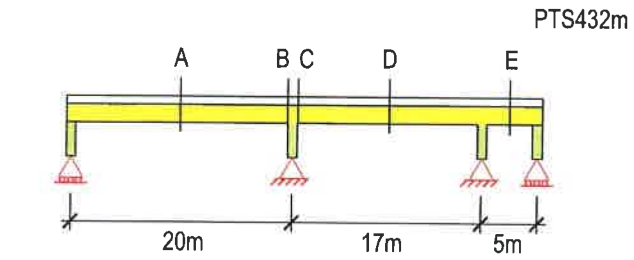
❖ [EC2, TR43]
Frequent load condition: 1*DL + 0.5*LL + 1*PT
Quasi-permanent load condition: 1*DL + 0.3*LL + 1*PT

For serviceability, the actions from the balanced loads from post-tensioning (PT) are used. These are due to "balanced loading." The background for this is explained in detail in reference [Aalami, 1990].

7.2 Stress Check

Critical Locations for Stress Check:

For hand calculation, the critical locations for stress check are selected based on engineering judgment. The selected locations may or may not coincide with the locations of maximum stress levels. This will introduce a certain degree of approximation in design, which reflects the common practice for hand calculations. Computer solutions generally calculate stresses at multiple locations along a span, thus providing greater accuracy.



Locations Selected for Detailed Design

FIGURE 7.2-1

By inspection, locations marked in Fig. 7.2-1 as sections A through E are considered critical for design. These are the midspan locations and the face-of-support locations of the first interior column.

24 ACI-318 specifies a "sustained" load case, but does not stipulate the fraction of live load to be considered "sustained." It is left to the judgment of the design engineer to determine the applicable fraction. The fraction selected varies between 0.2 and 0.5. The most commonly used fraction is 0.3, as it is adopted in this design example.

The moment diagrams due to the combined action of dead and live loading (Fig. 5-2 and 5-3) and the moment distribution due to post-tensioning (Fig. 6-5-1) are used to determine the design values at the selected locations.

Stresses:
$\sigma = (M_D + M_L + M_{PT})/S + P/A$
$S = I/Y_c$

Where, M_D, M_L and M_{PT} are the moments across the entire tributary of the design strip. S is the section modulus of the cross-sectional area reduced through effective width defined for bending action; A is the area of the entire tributary; I is the second moment of area of the portion of the cross-section that is defined by the effective width for bending; and Y_c is the distance of the centroid of the reduced section (defined for bending) to the farthest tension fiber of the section.

Y_T = 248 mm ; Y_B = 512 mm
S_{top} = 3.185e+10/248 = 1.284e+8 mm³
S_{bot} = 3.185e+10/512 = 6.221e+7 mm³
A = 917100 mm²
P/A = -1428 *1000/917100 = -1.56 MPa

A. Based on ACI 318-11/IBC 2012

Stress checks are performed for the two load conditions of total load and sustained load. For allowable values see Section 4.3(A).

Point A

❖ Total load combination
$\sigma = (M_D + M_L + M_{PT})/S + P/A$
$M_D + M_L + M_{PT}$ = (636+330.10 – 434.80)
= 531.30 kN-m

Top
σ = -531.30 *1000²/1.284e+8 – 1.56 = -5.70 MPa Compression < -16.80 MPa OK

Bottom
σ = 531.30 *1000²/6.221e+7 – 1.56 = 6.98 MPa Tension > 5.29 MPa NG

The stress check is considered acceptable, since it refers to "total" load condition. The member will be considered in "cracked" regime. Deflections have to be calculated using cracked sections

❖ Sustained load combination
$\sigma = (M_D + 0.3M_L + M_{PT})/S + P/A$
$M_D + 0.3\ M_L + M_{PT}$ = (636+0.3*330.10 – 434.80)
= 300.23 kN-m

Top
σ = -300.23*1000²/1.284e+8 – 1.56
= -3.90 MPa Compression < -12.60 MPa OK

Bottom
σ = 300.23 *1000²/6.221e+7 – 1.56
= 3.27 MPa Tension < 3.28 MPa OK

Since the tensile stress does not exceed the threshold of "Transition," the section is treated as uncracked. Otherwise deflections have to be calculated using cracked sections

B. Based on EC2

Stress checks are performed for the two load conditions of frequent load and quasi-permanent load. The outcome will determine whether crack width needs to be controlled or not. See section on "Allowable Stresses."

Point A

❖ Frequent load condition
$\sigma = (M_D + 0.5\ M_L + M_{PT})/S + P/A$
Stress thresholds:
Compression = -16.80 Mpa ; Tension = 2.77 MPa
$M_D + 0.5M_L + M_{PT}$ = (636 + 0. 5*330.10 – 434.80)
= 366.25 kN-m

Top
σ = -366.25 *1000²/1.284e+8 – 1.56 = -4.41 MPa Compression < -16.80 MPa OK

Bottom
σ = 366.25 *1000²/6.221e+7 – 1.56 = 4.32 MPa Tension > 2.77 MPa Control cracking

❖ Quasi-permanent load condition
$\sigma = (M_D + 0.3\ M_L + M_{PT})/S + P/A$
Stress thresholds:
Compression = - 12.60 Mpa; Tension = 2.77 MPa
$M_D + 0.3M_L + M_{PT}$ = (636+0. 3*330.10 – 434.80)
= 300.23 kN-m

Top
σ = -300.23*1000²/1.284e+8 – 1.56 = -3.90 MPa Compression < -12.60 MPa OK

Bottom
σ = 300.23 *1000²/6.221e+7 – 1.56
= 3.27 MPa Tension > 2.77 MPa
Hence control cracking[25]

C - Based on TR-43

For stress limits see Section 4.3(C) Design is based on 0.2mm crack width

25 EN 1992-1-1:2004(E) , Section 7.3.4

TABLE 7.2-1. Sevice Extreme Fiber Stresses at Selected Points (T133)

Load Combination		Point A	Point B	Point C	Point D	Point E
Based on ACI 08/IBC 2009						
Sustained load	f_t (MPa)	-3.90	2.16	1.81	-2.61	-0.86
	f_b (MPa)	3.27	-9.24	-8.51	1.80	-1.62
	F_t (MPa)	-12.60	3.97	3.97	-12.60	-12.60
	F_b (MPa)	5.29	-12.60	-12.60	5.29	-12.60
		OK	OK	OK	OK	OK
Total Load	f_t (MPa)	-5.70	4.78	4.08	-3.35	-0.54
	f_b (MPa)	6.98	-14.64	-13.20	3.34	-2.08
	F_t (MPa)	-16.80	3.97	3.97	-16.80	-16.80
	F_b (MPa)	5.29	-16.80	-16.80	5.29	-16.80
		NG*	NG*	NG*	OK	OK
Based on EC2						
Frequent Load	f_t (MPa)	-4.41	2.91	2.46	-2.82	-0.77
	f_b (MPa)	4.33	-10.78	-9.85	2.24	-1.75
	F_t (MPa)	-16.80	2.77	2.77	-16.80	-16.80
	F_b (MPa)	2.77	-16.80	-16.80	2.77	-16.80
		NG	NG	OK	OK	OK
Quasi-Permanent Load	f_t (MPa)	-3.90	2.16	1.81	-2.61	-0.86
	f_b (MPa)	3.27	-9.24	-8.51	1.80	-1.62
	F_t (MPa)	-12.60	2.77	2.77	-12.60	-12.60
	F_b (MPa)	2.77	-12.60	-12.60	2.77	-12.60
		NG	OK	OK	OK	OK
Based on TR-43						
Frequent Load	f_t (MPa)	-4.41	2.91	2.46	-2.82	-0.77
	f_b (MPa)	4.33	-10.78	-9.85	2.24	-1.75
	F_t – Unbonded	-16.80	3.74	3.74	-16.80	-16.80
	Bonded (MPa)	-16.80	4.57	4.57	-16.80	-16.80
	F_b – Unbonded	3.74	-16.80	-16.80	3.74	-16.80
	Bonded (MPa)	4.57	-16.80	-16.80	4.57	-16.80
		NG	OK	OK	OK	OK
Quasi-Permanent Load	f_t (MPa)	-3.90	NA	NA	-2.61	-0.86
	f_b (MPa)	NA	-9.24	-8.51	NA	-1.62
	(MPa)	-12.60	-12.60	-12.60	-12.60	-12.60
		OK	OK	OK	OK	OK

At point A

❖ Frequent load condition

$\sigma = (M_D + 0.5\ M_L + M_{PT})/S + P/A$

Stress limits

Unbonded tendons

Compression = -16.80 Mpa; Tension = 3.74 MPa

Bonded tendons

Compression = -16.80 Mpa

Tension for 0.2mm crack width; without bonded reinforcement = 4.57 Mpa

0.2mm crack witdth with bonded reinforcement =8.40 MPa

$M_D + 0.5M_L + M_{PT}$
= (636+0. 5*330.10 – 434.80) = 366.25 kN-m

Top

σ = -366.25 *1000²/1.284e+8 – 1.56 = -4.41 MPa Compression < -16.80 MPa OK

Bottom

σ = 366.25 *1000²/6.221e+7 – 1.56 = 4.32 MPa Tension > 3.74 MPa for unbonded tendon. Need to add non-prestressed reinforcment and control crack width.

Tension = 4.32 MPa< 4.57 MPa
OK for bonded tendon

❖ Quasi-permanent load condition

$\sigma = (M_D + 0.3\ M_L + M_{PT})/S + P/A$

$M_D + 0.3M_L + M_{PT}$ = (636+0. 3*330.10 – 434.80)
= 300.23 kN-m

Top

σ = -300.23*1000²/1.284e+8 – 1.56
= -3.90 MPa Compression < -12.60 MPa OK

Since the tensile stresses at one or more locations exceed the threshold for uncracked sections, rebar has to be provided in order to limit the crack width. For ACI-318 code, if the tensile stress exceeds the limit of 1√f'c, deflections should be calculated using cracked sections.

7.3 Crack Width Control

A. Based on ACI 318-11/IBC 2012

Since the tensile stress exceeds the limit for cracked sections, the section should be treated as cracked and the deflection should be calculated for cracked sections.

B. Based on EC2[26]

The allowable crack width for members reinforced with unbonded tendons (quasi-permanent load combination) is 0.3 mm, and for bonded tendon (frequent load combination) is 0.2 mm. Since in this example the maximum computed tensile stress exceeds the threshold limit, crack width calculation is required based on section 7.3.4 of EC2 code. If the calculated crack width exceeds the threshold, EC2 recommends to limit the bar diameter and bar spacing to the values given in Table 7.2N or 7.3N of EC2 to limit the width of cracks

Using EC2, The crack width calculation for frequent load combination is explained in the following.

Point A

Crack width, $W_k = S_{r,\max}\ (\varepsilon_{sm} - \varepsilon_{cm})$[27]

$\varepsilon_{sm} - \varepsilon_{cm} = [\sigma_s - k_t\ {}^*(f_{ct,eff}/\rho_{p,eff})(1 + \alpha_e\ \rho_{p,eff})]/E_s$
$\geq 0.6\ \sigma_s/E_s$

Where,

$\alpha_e = E_s/E_{cm}$ = 200000/32308 = 6.19

$\rho_{p,eff} = (A_s + \xi_1^2\ A'_p)/\ A_{c,eff}$

A_s = 0 mm²

A'_p = area of tendons within $A_{c,eff}$
= 12 *99 = 1188 mm²

$\xi_1 = \sqrt{}\ (\xi^*\ \varphi_s/\varphi_p)$

ξ = 0.5 (From Table 6.2)

φ_s = largest diameter of bar =22 mm

φ_p = 1.75* 13 = 23 mm

$\xi_1 = \sqrt{}$ (0.5* 22/23) = 0.70

$A_{c,eff} = h_{c,eff}{}^*bw$

$h_{c,eff}$ = lesser of (2.5*(h-d), (h-x)/3 , (h/2))
= 4.33* 760/(4.33+4.41) = 377 mm

d = 760- 40-22/2 = 709 mm

$h_{c,eff}$ = lesser of (2.5*(760-709), (760-377)/3 , (760/2)) = 128 mm

$A_{c,eff}$ = 128*460 =58880 mm²

$\rho_{p,eff}$ = (0 + 0.70²*1188)/ 58880 =0.00989

$\sigma_s = (f/E_c){}^*E_s$

f = tensile stress due to DL+0.3LL = $(M_D+0.3\ M_L)/S$
= (636+0.5*330.10)*1000²/6.22*10⁷
= 12.88 MPa

σ_s = (12.88/32308)*200000 =79.73 MPa

k_t = 0.4 (coefficient for long-term loading)

$f_{ct,eff} = f_{ctm}$ = 0.3 *(28)^(2/3) = 2.77 MPa

$\varepsilon_{sm} - \varepsilon_{cm} = [\sigma_s - k_t\ {}^*(f_{ct,eff}/\rho_{p,eff})(1 + \alpha_e\ \rho_{p,eff})]/Es$
= [79.73– 0.4 *(2.77/0.00989)(1 + 6.19*0.00989)]/200000
= -0. 000196 < 0.6*79.73/200000
= 0.000239 sr,max = 1.3*(h-x) =1.3* 383 = 498 mm

Crack width, W_k = 498*0.000239 = 0.12 mm
< 0.2 mm OK

Provide minimum reinforcement for cracking. It is provided with minimum rebar in 7.3.4.

26 EN 1992-1-1:2004(E) , Section 7.3.3

27 EN 1992-1-1:2004(E) , Section 7.3.4

Similarly the crack width calculation should be performed at B.

EXAMPLE 1

To illustrate the procedure for crack control by way of addition of reinforcement, as recommended in EC2, as an example let the maximum tensile stress exceed the threshold value by a large margin.

Given: computed hypothetical farthest fiber tensile stress in concrete f = 20MPa

Required: reinforcement design for crack control
Calculate stress in steel at location of maximum concrete stress: σ_s = (f/Ec)*Es
Where f is the hypothetical tensile stress in concrete at crack tip

σ_s = (20/32308)*200000 =123.81 MPa
(this is a hypothetical value)
Crack spacing can be limited by either restricting the bar diameter and/or bar spacing. Use the maximum bar spacing from Table 7.3 N for the σ_s of 123.81 MPa.
From Table, for 160 MPa - 300 mm

Since it is less than the minimum steel stress, use the same spacing as 160 MPa. The maximum spacing for 12381 MPa is 300 mm. Note that based on the magnitude of the computed tensile stress in concrete the area of the required reinforcement for crack control is calculated separately.

C. Based on TR-43[28]

The allowable crack width for all members is 0.2 mm. Since in this example the maximum computed tensile stress at A exceeds the threshold limit for unbonded tendon, crack width calculation is required based on EC2 section 7.3.4. From the crack width calculation performed at A for the EC2 code in *B* of this section, it is found that calculated crack width, 0.12 mm, is less than the allowable width of 0.2mm. If the calculated crack width exceeds the threshold, TR43 recommends either revise the design parameters (slab depth, prestress level etc) or add additional bonded reinforcement and recalculate crack width.

For bonded tendons, if the hypothetical tensile stress exceeds the threshold values, rebar needs to be added to limit the cracking as follows:

Add 0.0025A_t rebar in tension zone as close to extreme tension fiber as practical for every 1MPa of stress above the threshold up to the stress of $0.30f_{ck}$. The addition of this rebar for overage of stress is deemed to satisfy the intent of crack control.

Since the maximum computed tensile stresses at the selected points are below the threshold for crack control, added rebar is not required. For completeness, the following example illustrates the procedure, should crack control become necessary.

EXAMPLE

To illustrate the procedure for crack control rebar for bonded tendon, as an example let the maximum computed tensile stresses exceed the threshold value.

Given: Concrete strength: 28 MPa; threshold for crack control 3.5 MPa; computed hypothetical stresses:
Top fiber: f_t = -4.41 MPa compression
Bottom fiber: f_b = 4.33 MPa tension
Maximum allowable tension = $0.3f_{ck}$ = 8.4 MPa

Required: Determine (i) depth of neutral; (ii) area of tension zone; (iii) percentage of rebar to be added using the area of tension zone.
Rebar to be added: A_s = 0.0025*A_t*(4.33-3.5)
Where, A_t = area of tension zone
Depth of neutral axis
x = 4.33* 760/(4.33+4.41) = 377 mm
A_t = 377*460 = 1.734e+05 mm^2
A_s = 0.0025*1.734e+05*(4.33-3.5) = 360 mm^2
No.of Bars = 360 /387 = 0.93 Use 1- 22 mm bars
A_s = 1 *387 = 387 mm^2

7.4 Minimum Reinforcement

There are several reasons why the building codes specify a minimum reinforcement for prestressed members. These are:

Crack control: Bonded reinforcement contributes in reducing the width of local cracks. The contribution of bonded reinforcement to crack control is gauged by the strain it develops under service load. The force developed by bonded reinforcement in resisting cracking depends on the area of steel and its modulus of elasticity.

The area of reinforcement considered available for crack control is $(A_s + A_{ps})$, where A_{ps} is the area of bonded tendons. It is recognized that both bonded and unbonded prestressing provide precompression. While the physical presence of an unbonded tendon may not contribute to crack control, the contribution through the precompression it provides does. However, for code compliance and conformance with practice, the contribution of unbonded tendons is not included in the aforementioned sum.

Ductility: An underlying reason of ACI-318 requirement of minimum bonded reinforcement for members reinforced with unbonded tendons is to enhance ductility at ULS. Current ACI-318/IBC does not specify a minimum of non-stressed bonded reinforcement for members reinforced with bonded tendons.

Use 22 mm bars (Area = 387 mm^2; Diameter = 22 mm) for top and bottom, where required
d = 760 – 40 - 22/2 = 709 mm

A. Based on ACI 318-11/IBC 2012[29]

❖ Unbonded Tendon
Minimum required, top
A_s = 0.004*A_{tens}
A_{tens} is the area of the section between the tension fiber and the section centroid. The minimum rebar is required for members reinforced with unbonded tendons. The added rebar is to reduce the in-service crack width and enhance the ductility of the member for ultimate strength condition. Since the minimum rebar is intended to address the flexural performance of the member, the cross-sectional properties associated with the flexure are used for the determination of its area.

Top bars at supports 1, 2 and 3
A_s = 0.004*[125*2460 + (248 – 125)*460]
= 1457 mm^2
Number of Bars = 1457/387 = 3.76;
Use 4 - 22 mm bars; A_s= 4 *387 = 1548 mm^2 OK
Top bar at support 4
A_s = 0.004*[125*1250 + (310 – 125)*460]
= 966 mm^2
Number of Bars = 966/387 = 2.49;
Use 3 - 22 mm bars; A_s = 3 *387 =1161 mm^2 OK
Minimum required at bottom for spans 1 and 2:
A_s = 0.004*A_{Tens}= 0.004*(460*512) = 942mm^2
Number of Bars = 942/387 =2.43;
Use 3-22 mm bars; A_s = 3*387 = 1161 mm^2 OK
Minimum required at bottom for span 3
A_s = 0.004*(460*450) = 828 mm^2
Number of Bars = 828/387 = 2.14;
Use 3-22 mm bars; A_s = 3*387 = 1161 mm^2

Since at midspan, the tension at service condition is at the top fiber, the minimum reinforcement calculated for the top will be used. In this case, 3-22mm will be adequate. Hence
A_s = 1161 mm^2; use 3-22 mm bars at top of midspan

❖ Bonded (grouted) tendons
There is no requirement for minimum reinforcement based on either geometry of the design strip, nor its hypothetical tensile stresses. The minimum requirement is handled through the relationship between the cracking moment of a section and its nominal strength in bending. This is handled in the "strength" check of the member (section 8 of this example). The code check for strength adequacy after the initiation of first crack is handled in the strength design (ULS).

B. Based on EC2[30]

EC2 specifies the same requirement for minimum reinforcement at supports and spans, and also for both unbonded and bonded tendons. Two checks apply. One is based on the cross-sectional geometry of the design strip and its material properties and the other on computed stresses. In the former, the minimum reinforcement applies to the combined contributions of prestressed and non-prestressed reinforcement. Hence, the participation of each is based according to the strength it provides, the prestressing steel is accounted for with higher values. The reinforcement requirement for crack control is handled separately.

❖ Unbonded and bonded tendons
Spans
$A_{smin} \geq (0.26^* f_{ctm} {}^*b_t{}^*d/f_{yk}) \geq 0.0013^* b_t {}^*d$
b_t = 460 mm
d = 760- 40-22/2 = 709 mm
f_{ctm} = 0.3 *28$^{(2/3)}$ = 2.77 MPa
(i) A_s = 0.26* f_{ctm}*b_t*d/f_{yk}
= 0.26*2.77* 460*709/460 = 511 mm^2
(ii) A_s = 0.0013*b_t*d = 0.0013 *460*709
= 424 mm^2
Therefore, A_s = 511 mm^2
Contribution of reinforcement from bonded prestressing
Point A
A_{ps} *(f_{pk}/f_{yk}) = 12* 99*1860/460
= 4804 mm^2 > 511mm^2
Points D & E:,
A_{ps} *(f_{pk}/f_{yk}) = 9* 99*1860/460
= 3603 mm^2 > 511mm^2
Hence, no additional bonded reinforcement is required.

---

28 TR-43, Second edition, Section 5.8.1 and 5.8.3

29 ACI 318-11, Section 18.9

30 EN 1992-1-1:2004(E) , Section 9.2.1 and 7.3.2

Supports

$A_{smin} \geq (0.26^* f_{ctm} {}^*b_t{}^*d/f_{yk}) \geq 0.0013^* b_t {}^*d$

b_t = mean width of the tension zone

depth of tension zone, (h-c) (Refer Fig.7.4-1)

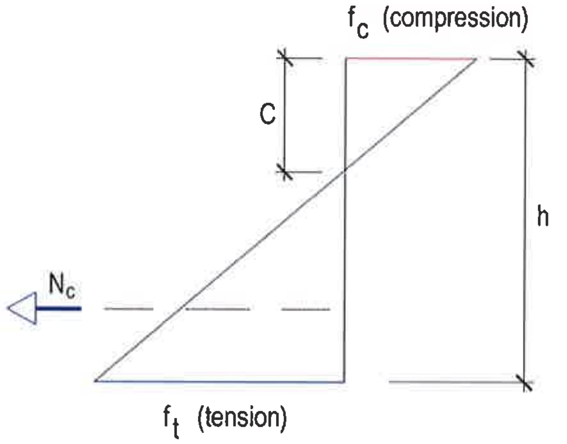


FIGURE 7.4-1 Distribution of Stress over Section

= 2.91*760/ (2.91+10.78)

= 162 mm (Consider point B)

b_t = [2460*125+ 460*(162-125)]/162 = 2003 mm

d = 760- 40-22/2 = 709 mm

(i) $A_s = 0.26^* f_{ctm}{}^*b_t{}^*d/f_{yk}$

= 0.26*2.77* 2003*709/460 = 2223 mm^2

(ii) $A_s = 0.0013^*b_t{}^*d$ = 0.0013 *2003*709

= 1846 mm^2 Therefore, A_s = 2223 mm^2

Contribution of reinforcement from bonded Prestressing:

$A_{ps}{}^*(f_{pk}/f_{yk})$ = 12* 99*1860/460

= 4804 mm^2 > 2223mm^2

Hence, no additional bonded reinforcement is required.

❖ Minimum reinforcement for crack control

In EC2 necessity of reinforcement for crack control is triggered, where computed tensile stresses exceed a code-specified threshold.

Since the hypothetical tensile stress of concrete exceeds the threshold for crack control at point A, cracking reinforcement need to be provided.

At point A

$A_{smin} = k_c\, k\, f_{ct,eff}\, A_{ct} / \sigma_s$

A_{ct} is the area of the concrete section in tension zone.

f_t = -4.41 Mpa (compression at top)

f_b = 4.33 MPa (tension at bottom)

$\sigma_s = f_{yk}$ = 460 MPa

$f_{ct,eff} = f_{ctm}$ = 0.3 *(28)$^{(2/3)}$ = 2.77 MPa

k = 0.678 (interpolated for h=760 mm)

(h – c) = 4.33*760/ (4.41+4.33) = 377 mm

A_{ct} = 377*460 =173420 mm^2

$k_c = 0.4^*\ [1\text{-}(\ \sigma_c /(\ k_1\ (h/h^*)\ f_{ct,eff})]$

$\sigma_c = N_{ED}$ /bh = 1.56 MPa

h* = 760 mm

k_1 = 1.5

k_c = 0.4* [1-(1.56 /(1.5 (760/760*) 2.77)] = 0.25

A_{smin} = 0.25* 0.678* 2.77* 173420 /460 = 177 mm^2

Provide one 22mm bar ($A_{s,prov}$ = 1* 387 =387 mm^2)

$A_{smin,crack}$ = 387 mm^2

At point B

$A_{smin} = k_c\, k\, f_{ct,eff}\, A_{ct} / \sigma_s$

A_{ct} is the area of the concrete section in tension zone.

f_t = 2.91 Mpa (tension at top, refer to Fig. 7.4-1)

f_b = -10.78 MPa (compression at bottom)

PTS115

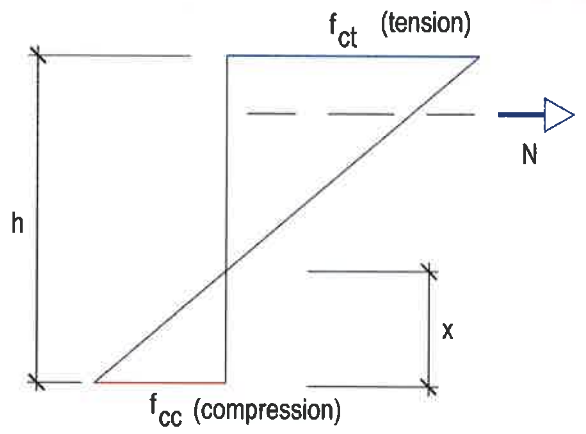


FIGURE 7.4-2 Distribution of Stress over Section

$\sigma_s = f_{yk}$ = 460 MPa

$f_{ct,eff} = f_{ctm}$ = 0.3 *(28)$^{(2/3)}$ = 2.77 MPa

k = 0.678 (interpolated for h=760 mm)

Distance of neutral axis from top

= 2.91*760/ (2.91+10.78) = 162 mm

A_{ct} = 162*2460 =398520 mm^2

$k = 0.4^*\ [1\text{-}(\ \sigma_c /(\ k_1\ (h/h^*)\ f_{ct,eff})]$

$\sigma_c = N_{ED}$ /bh = 1.56 MPa

h* = 760 mm

k_1 = 1.5

k_c = 0.4* [1-(1.56 /(1.5 (760/760*) 2.77)]= 0.25

A_{smin} = 0.25* 0.678* 2.77* 398520 /460= 407 mm^2

Provide 2-22mm bar ($A_{s,prov}$ = 2* 387 =774 mm^2)

$A_{smin,crack}$ = 774 mm^2

C. Based on TR-43

❖ Unbonded Tendon

(i) Flexural un-tensioned reinforcement[31]

[31] TR-43 2nd Edition, Section 5.8.7

TABLE 7.4-2 Summary of Minimum Rebar (mm^2) (T1345I)

Code	Unbonded				Bonded			
	Point A	Point B & C	Point D	Point E	Point A	Point B & C	Point D	Point E
ACI/IBC	942	1457	942	966	0	0	0	0
EC2	177	407	0	0	177	407	0	0
TR43	1303	2017	603	0	0	0	0	0

TR-43 stipulates that a minimum amount of non-prestressed bonded reinforcement be present at all locations for the full tension force generated by the computed tensile stresses in the concrete under service load combination.

At point A:(Frequent load combination)

Depth of tension zone: h-x = $-f_{ct}{}^*h/(f_{cc}-f_{ct})$

Refer to Fig. 7.4-2 where f_{cc} is the concrete fiber stress in compression; f_{ct} is the extreme concrete fiber stress in tension.

f_{ct} = tensile stress (-ve) = -4.33 MPa

f_{cc} = compressive stress = 4.41 MPa

h = depth of the section = 760 mm

b = width of the section = 460 mm

x = depth of the compression zone

h-x = -(-4.33)*760/(4.41+4.33) = 376 mm

$A_s = F_t /(5^*f_{yk} /8)$

Where F_t is the total tensile force over the tensile zone of the entire section

$F_t = -f_{ct}{}^*b^*(h\text{-}x)/2$ = -(-4.33)*460 *376/(2*1000)

= 374.46 kN

A_s = 374.46*1000/(5*460/8) =1303 mm^2

Provide 4- 22mm bar ($A_{s,prov}$ = 4* 387 =1548 mm^2)

At points B and C

Depth of tension zone: h-x= $-f_{ct}{}^*h/(f_{cc}-f_{ct})$

Refer to Fig. 7.4-2 where f_{cc} is the concrete fiber stress in compression; f_{ct} is the extreme concrete fiber stress in tension.

f_{ct} = tensile stress (-ve) = -2.91

f_{cc} = compressive stress= 10.78 MPa

h = depth of the section = 760 mm

b = width of the section = 2460 mm

x = depth of the compression zone

h-x= -(-2.91)*760/(2.91+10.78) = 162 mm

$A_s = F_t /(5^*f_{yk} /8)$

Where F_t is the total tensile force over the tensile zone of the entire section

$F_t = -f_{ct}{}^*b^*(h\text{-}x)/2$ = -(-2.91)*2460 *162/(2*1000) = 579.85 kN

A_s = 579.85*1000/(5*460/8) =2017 mm^2

Provide 6- 22mm bar ($A_{s,prov}$ = 6* 387 =2322 mm^2)

At point D

f_{ct} = tensile stress (-ve) = -2.24 MPa f_{cc}

= compressive stress=2.82 MPa

h = 760 mm ; b =460 mm

h-x = -(-2.24)*760/(2.24+2.82) = 336 mm

$A_s = F_t /(5^*f_{yk} /8)$

$F_t = -f_{ct}{}^*b^*(h\text{-}x)/2$ = -(-2.24)*460 *336/(2*1000)

= 173.11 kN

A_s = 173.11 *1000/(5*460/8) =603 mm^2

Provide 2- 22mm bar (As,prov = 2* 387 =774 mm^2)

❖ Other minimum reinforcement for unbonded tendons.

In addition to the preceding that was based on the value of hypothetical tensile stress, TR-43 requires that the bonded reinforcement at each section not to be less that that specified in EC2[32]. The computation of the minimum rebar based on EC2 is carried out in the above section under EC2.

The second check for minimum bonded reinforcement is the same as carried out for EC2 code in the preceding section.

❖ Bonded Tendon

There are no minimum rebar requirements for one-way spanning members reinforced with bonded tendons. Any additional SLS reinforcement will be related to the design crack width, and the potential of the cracks exceeding the design value. This check was performed in Section 7.3(B).

The minimum area of rebar required using the codes covered is summarized in Table 7.4-1

[32] TR-43 2nd Edition, Section 5.8.8

7.5 Deflection Check

Recognizing that (i) the accurate determination of probable deflection is complex [see Chapter 4, Section 4.10.6]; and (ii) once a value is determined, the judgment on its adequacy at design time is subjective, and depends on unknown, yet important, parameters such as age of concrete at time of installation of nonstructural members that are likely to be damaged from large displacement. For hand calculation deflection checks are generally based on simplified procedures. A rigorous analysis is initiated, only where the parameters of design and applied loads are more reliably known. In most cases, post-tensioned members are sized according to recommended span/depth ratios proven to perform well in deflection.[33]

The simplified procedure includes:
(i) For visual and functional effects, total long-term deflection from the day the supports are removed not to exceed span/250 for EC2 or span/240 ACI-318. Camber can be used to offset the impact of displacement.
(ii) Immediate deflection under design live load not to exceed span/500 for EC2 or span/480 for ACI-318. Both ACI 318/IBC and EC2, tie the deflection adequacy to displacement subsequent to the installation of members that are likely to be damaged. This requires knowledge of construction schedule and release of structure for service. In the following the common design practice is followed.

For assessment of long-term displacement in the context of foregoing, ACI-318 recommends a multiplier factor of 2[34].

The deflections are calculated using a frame analysis program for each of the load cases: dead, live, and post-tensioning. Gross cross-sectional area and linear elastic relationships are used. Since the stress level for which the design was carried out falls essentially in the transition regime, the elastically calculated deflections must be adjusted to allow for cracking at locations where cracking stresses are exceed the threshold of "uncracked" regime. Strictly speaking, a cracked deflection calculation has to be performed,[35] where stresses exceed the "transition" regime. However, for hand calculation, recognizing that the locations of probable cracks, as in this example are few, the option of "magnifying" elastic deformation by a factor that allows for cracking is used.

The critical location is in span 1. The values for span 1 from the frame analysis are:

Span 1 Deflection	
Dead Load	27.8 mm
Post-Tensioning	-19.2 mm
Dead Load + PT	8.6 mm
Live load deflection	14.5 mm

The maximum stress under total loading at midspan is 6.98 MPa. Since this is greater than 0.62 $\sqrt{f'_c}$, adjustment to the calculated deflection is required.

There are several options available to adjust elastically calculated deflection values, if the computed tensile stresses exceed cracking. Among the most commonly used are: (i) substitution of the gross moment of inertia (I_g), by an equivalent moment of inertia (I_e), followed by the magnification of the elastically calculated deflection by the ratio of (I_g/I_e); and (ii) use of a bilinear deflection calculation, in which the amount of deflection prior to cracking is calculated using Ig and the elastic solution, the deflection after the initiation of crack is calculated for the overage of load, using the cracked moment of inertia (I_{cr}).

For prestressed sections the equivalent moment of inertia is calculated using the following relationship [PTI design manual, 1990].

$I_e = [1 - 0.30*(f_{max} - 0.5\sqrt{f'_c})/ 0.5\sqrt{f'_c}]*I_g$
f'_c is in MPa

Where, I_e is the effective moment of inertia; I_g is the moment of inertia based on the gross cross-sectional area. Initially, the relationship was proposed for f_{max} not exceeding $\sqrt{f'_c}$. But, it is now used for values above $\sqrt{f'_c}$.

The calculated maximum tensile stress
$f_{max} = 6.98$ MPa.
Reduction in moment of inertia due to cracking:
$I_e = [1 - 0.30*(f_{max} - 0.5\sqrt{f'_c})/0.5\sqrt{f'_c}]*I_g$
$= [1 - 0.30*(6.98\text{-}2.65)/2.65]*I_g$
$= 0.51*I_g$
Hence deflection due to dead load and PT
= 8.6/0.51 = 16.86 mm
Live load deflection with cracking allowance
= 14.5/0.51 = 28.43 mm

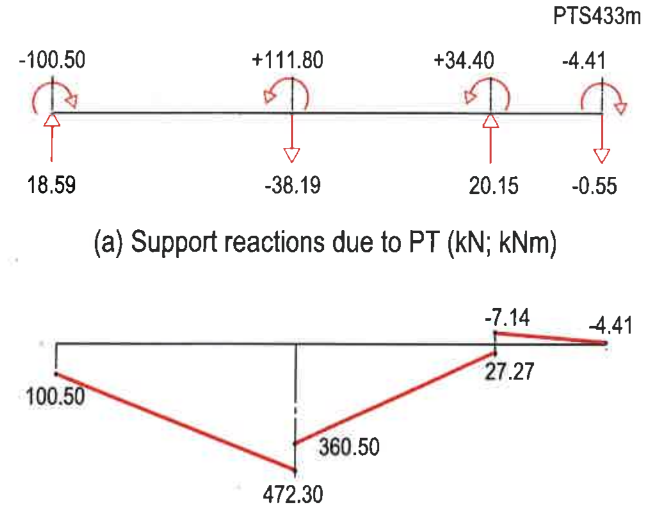
33 TR-43 2nd Edition, Section 5.8.4; ADAPT-TN292
34 ACI multiplier 2
35 Compter programs, such as ADAPT Floor do cracked deflection calculation

❖ Long-term deflection
Multiplier factor assumed for effects of creep and shrinkage on long-term deflection = 2[36]
Load combination for long-term deflection, using a factor of 0.3 for sustained "quasi-permanent" live load :

(1.0*DL + 1.0*PT + 0.3*LL)*(1 + 2)

Long-term deflection: (1 + 2)*(16.86 + 0.3*28.43) = 76.17 mm
Deflection ratio = 76.17/(20000) = 1/263 < 1/250 OK
Instantaneous deflection due to design live load
Live load deflection = 28.43 mm.
Deflection ratio = 28.43/(20,000) = 1/703 OK

Deflection does not generally govern the design for members dimensioned within the limits of the recommended values in ACI 318 and balanced with post-tensioning tendons within the recommended range [Aalami et al, 2003], and when subject to loading common in building construction. For such cases, deflections are almost always within the permissible code values, when design is performed within U or T stress values.



(a) Support reactions due to PT (kN; kNm)

(b) Hyperstatic moment distribution (kNm)

Hyperstatic (Secondary) Actions

FIGURE 8.2-1

8. CODE CHECK FOR STRENGTH

8.1 Load Combinations

(i) ACI-318/IBC
1.2*DL + 1.6*LL + 1*HYP
1.4*DL + 1*HYP
(ii) EC2
1.35*DL + 1.5*LL + 1*Hyp
(iii) TR43
1.35*DL + 1.5*LL + 0.9*Hyp

36 ACI- 318 multiplier factor

For strength combination, the hyperstatic (Hyp) actions (secondary) due to prestressing are used. The background for this is explained in detail in Chapter 4, Section 4.11.2.

8.2 Determination of Hyperstatic Actions

The hyperstatic moments are calculated from the reactions of the frame analysis under balanced loads from prestressing (Loads shown in Fig. 6.4-2). The reactions obtained from a standard frame analysis are shown in Fig. 8.2-1(a). The reactions shown produce hyperstatic moments in the frame as shown in Fig. 8.2-1(b).

The hyperstatic (secondary) reactions must be in self-equilibrium, since the applied loads (balanced loads) are in self-equilibrium.

Check the validity of the solution for static equilibrium of the hyperstatic actions using the reactions shown in Fig. 8.2-1a.
ΣVertical Forces
= 18.59 – 38.19 +20.15 – 0.545 =0.005 OK
ΣMoments about Support 1
= -100.50+ 111.80+34.40 – 4.41 –(38.19*20) + (20.15*37) – (0.55*42) = -0.06 ≈ 0 OK

Support reactions due to post-tensioning are applied to the beam in order to construct the hyperstatic moment diagram shown 8.2-1(b). The support reactions are shown in part (a) of the figure.

Reduce hyperstatic moments to face-of-support using linear interpolation.
For right face of support (FOS) of span 1:
$M_{HYP} = 472.30 - [(472.30 - 100.50)/20]*0.45/2$
$= 468.12$ kN-m

A number of engineers use the expression given below to compute hyperstatic (secondary) moments due to prestressing. This expression gives acceptable results for articulated members, only if the balanced loads used in the determination of post-tensioning moments (M_{pt}) satisfy equilibrium.

$M_{hyp} = M_{pt} - P\ e$

TABLE 8.3-1 Ultimate Design Moments (T135)

	Point A	Point B	Point C	Point D	Point E
M_D (kN-m)	636.00	-926.00	-802.80	262.40	-68.88
M_L (kN-m)	330.10	-480.70	-416.80	136.20	-35.75
M_{HYP} (k-ft)	286.40	468.10	356.10	193.90	-5.77
ACI 318-08/IBC 2009 : 1.2DL+1.6LL+1Hyp					
M_U (k-ft)	1577.76	-1412.22	-1274.14	726.70	-145.63
EC2 : 1.35DL+1.5LL+1Hyp					
M_U (k-ft)	1640.15	-1503.05	-1352.88	752.44	-152.38
TR 43 : 1.35DL+1.5LL+0.9Hyp					
M_U (k-ft)	1611.51	-1549.86	-1388.49	733.05	-151.81

Where M_{hyp} is the secondary moment, P is the post-tensioning force, and e is the eccentricity of the post-tensioning. The expression does not afford a validity check, such as the static equilibrium of all hyperstatic actions.

8.3 Calculation of Design Moments

The design moment (M_u) is the factored combination of dead, live and hyperstatic moments.

❖ Using ACI/IBC
Design moments are:
$M_{U1} = 1.2*M_D + 1.6*M_L + 1.0*M_{HYP}$
$M_{U2} = 1.4*M_D + 1.0*M_{HYP}$

The second combination governs, where dead loads are eight times or larger than live loads. This is rare. The moments shown in Fig. 8.2-1 are centerline moments. These are reduced to the face-of-support in Table 8.3-1.

By inspection, the second load combination does not govern, and will not be considered in the following.

The factored moment computed for EC2 and TR43 are listed in the following table.

8.4 Strength Design for Bending and Ductility

The strength design for bending consists of two provisions, namely

❖ The design capacity ($\Phi*M_n$; R) shall exceed the demand. A combination of prestressing and non-prestressed steel provides the design capacity

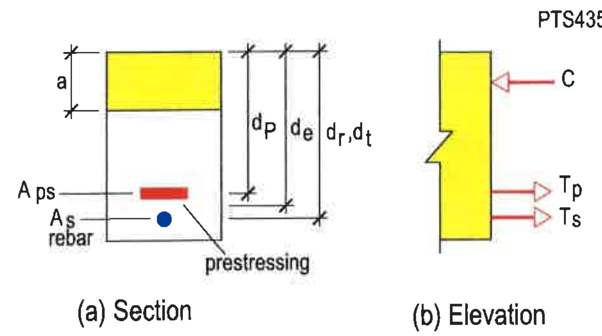
❖ The ductility of the section in bending shall not be less than the limit set in the associated building code. The required ductility is deemed satisfied, if failure of a section in bending is initiated in post-elastic response of its tension reinforcement, as opposed to crushing of concrete. For the codes covered in this example this is achieved through the limitation imposed on the depth of the compression zone (see Fig. C-8.4-1 in Chapter 6). The depth of compression zone is generally limited to 50% or less than the distance from the compression fiber to the farthest reinforcement (dr). Since the concrete strain (ϵ_c) at crushing is assumed between 0.003 and 0.0035, the increase in steel strain (ϵ_s) will at minimum be equal to that of concrete at the compression fiber. This will ensure extension of steel beyond its yield point (proof stress) and hence a ductile response.

For expeditious hand calculation, the flexural capacity of a post-tensioned member in common building structures can be approximated by assuming a conservative maximum stress for prestressing tendons. For detailed application of the code-proposed formulas refer to Chapter 12 for the calculation of a section's capacity both on the basis of strain compatibility and approximate code formulas. For daily hand calculation in the a consulting office, unless a software is used use the following simple procedure.

There are two justifications, why the simplified method for ULS design of post-tensioned sections in daily design work are recommended. These are:
(i) Unlike conventionally reinforced concrete, where at each section along a member non-prestressed reinforcement must be provided to resist the design moment, in prestressed members this may not be necessary. Prestressed members possess a base capacity along the entire length of prestressing tendons (Fig. C-8.4-2b in Chapter 6). Non-prestressed reinforcement is needed only at sections, where the moment demand exceeds the base capacity of the section.
(ii) In conventionally reinforced concrete, the stress used for rebar at ULS is well-defined in major building codes. For prestressed sections, however, the

TABLE 8.4-1 Summary of Reinforcement for Strength Limit State (T136SI)

Code	Unbonded				Bonded			
	Point A	Point B & C	Point D	Point E	Point A	Point B & C	Point D	Point E
ACI/IBC	2368	2631	1161	0	2463	2625	926	0
EC2	2915	3399(top) 1180(bot)	1134	0	2915	3399(top) 1180(bot)	1134	0
TR43	2703	3821(top) 1602(bot)	986	0	2836	3708(top) 1489(bot)	1134	0



Distribution of Basic Forces on a Rectangular Section

FIGURE C8.4-3

stress in tendon at ULS is oftentimes expressed in terms of an involved relationship—hence the tendency to use a simplified, but conservative scheme for everyday hand calculation. For repetitive work, computer programs are recommended.

Using strain compatibility procedure [37] the required reinforcement for each of the three codes are calculated. The outcome is listed in Table 8.4-1.

Using strain compatibility procedure[38] the required reinforcement for each of the three codes are calculated. The outcome is as follows:

❖ **Cracking moment larger than moment capacity:** Where cracking moment of a section is likely to exceed its nominal capacity in flexure, reinforcement is added to raise the moment capacity. In such cases, the contribution of each reinforcement is based on the strength it provides. If the minimum value is expressed in terms of cross-sectional area of reinforcement, the applicable value for this requirement is: $(A_s + A_{ps}*f_{py}/f_y)$.

[37] ADAPT-TN178
[38] ADAPT-TN178

TABLE 8.4-2 Cracking Moment Values and the Respective Data (T138SI)

Basic parameters and analysis		Section A	Section B&C	Section D
	S_{top} (mm³)		1.28e+8	
	S_{bot} (mm³)	6.22e+7		6.22e+7
	P (kN)	1428	1428	1071
	P / A (MPa)	-1.56	-1.56	-1.17
	fr + (P/A)	4.87	4.87	4.48
	Mcr (KNm)	302.91	623.36	278.66
	1.2 Mcr (kNm)	363.50	748.03	334.39
	Φ Mn (kNm)	1234.99	1234.99	637.63
	Status	OK	OK	OK

A. Based on ACI 318-11/IBC 2012

❖ Bonded (grouted) tendons
ACI 318 [39] /IBC stipulates that beams and one-way slabs reinforced with bonded tendons develop a nominal moment capacity at ULS not less than 1.2 times their cracking moment M_{cr}.

The necessity and amount of rebar is defined as a function of cracking moment of a section (M_{cr}). For pre ACI-318 Section 9.5.2.3 stressed members

$M_{cr} = (f_r + P/A)*S$
Where, fr is the modulus of rupture defined [40]
$f_r = 0.625 \sqrt{f'c} = 0.625 \sqrt{28} = 3.31$ MPa
P/A is the average precompression, and S the section modulus. The Table 8.4-2 summarizes the leading values and the outcome. The computation of the cracking moment and the nominal capacity are given in the following table.

Since at the selected sections, the design capacity of the section with prestressing alone exceeds $1.2*M_{cr}$, no additional rebar is required from this provision.

[39] ACI 318-11 Section 18.8.2
[40] ACI-318 Section 9.5.2.3

In design situations like above, where the design is initiated by determination of whether a value is less or more than a target, it is advisable to start the check using a simplified, but conservative procedure. If the computed value is closer to the target than the approximations in the simplified method, design check can be followed with a more rigorous computation.

Assume the following for simplified calculation:
Strand CGS = 70 mm
hence $d = h$ (thickness) - 70
Moment arm = 0.9d
Design force in strand = A_{ps}*1860 MPa ; Φ = 0.9
At midspan, with 12 strands
$\Phi *M_n$ = 0.9* 12*99 *1860*0.9 *(760 – 70)/10^6
= 1234.99 kNm

Design moment at other locations are calculated in a similar manner.

B. Based on EC2

❖ *Unbonded tendons*

EC2 [41] requires that for beams reinforced with unbonded tendons the total amount of prestressed and nonprestressed shall be adequate to develop a factored load at least 1.15 times the cracking load computed on the basis of the modulus of rupture of the section. In practice, this is taken as cracking moment of the section M_{cr}.

The necessity and amount of rebar is defined as a function of cracking moment of a section (M_{cr}). For prestressed members
$M_{cr} = (f_r + P/A)*S$
Where, fr is the modulus of rupture[42]
$f_r = f_{ctm} = 0.3f_{ck}^{(2/3)}$ = 2.77 MPa
P/A is the average precompression, and S the section modulus. The Table 8.4-3 summarizes the leading values and the outcome.

Since at the selected sections, the design capacity of the section with prestressing alone exceeds 1.15*M_{cr}, no additional rebar is required from this provision.

In design situations like above, where the design is initiated by determination of whether a value is less or more than a target, it is advisable to start the check using a simplified, but conservative procedure. If the

TABLE 8.4-3 Cracking Moment Values and the Respective Data for EC2

Basic parameters and analysis		Section A	Section B&C	Section D
	S_{top} (mm³)		1.28e+8	
	S_{bot} (mm³)	6.22e+7		6.22e+7
	P (kN)	1428	1428	1071
	P / A (MPa)	-1.56	-1.56	-1.17
	fr + (P/A)	4.33	4.33	3.94
	Mcr (KNm)	269.33	554.24	245.07
	1.15 Mcr (kNm)	309.73	637.38	281.83
	Φ Mn (kNm)	1193.23	1193.23	616.07
	Status	OK	OK	OK

computed value is close to the target, design check can be followed with a more rigorous computation.
Assume the following:
Cover to strand CGS = 70 mm;
hence $d = h$ (thickness) - 70
Moment arm = 0.9d
Design force in strand = A_{ps}*1860 MPa/1.15;
At midspan, with 12 strands, 1860 MPa strength
$\Phi *M_n$ = 12*99 *(1860/1.15)*0.9 *(760 – 70)/10^6 = 1193.23 kNm

Design moment at other locations are calculated in a similar manner.

Strength computations performed herein were limited to points considered critical by inspection. When spans and loading are not regular, the selection of critical points by inspection becomes difficult. In such cases, stress and strength checks must be performed at a greater number of locations. Also, note that due to the contribution of tendon to ultimate strength, and change in drape of tendon along the length of a member, the most critical location for design is not necessarily the location of maximum moment.

The envelope of total reinforcement is given in Table 8.4-4.

8.5 One-Way Shear Design

The shear design for the right support of span 1 will be followed in detail, since this is the most critical location. The procedure for the shear design of other locations is identical.

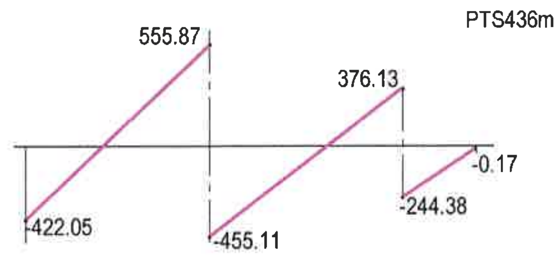
A. Based on ACI 318-11/IBC 2012

Distribution of design shear is shown in Fig. 8.5-1. The

[41] EN 1992-1-1:2004 (E), Section 9.2.1.1(4)
[42] EN 1992-1-1:2004 (E), Section 7.1(3). Here tensile stress limit for uncracked section is used.

TABLE 8.4-4 Envelope of Reinforcement for Serviceability (SLS) and Strength Conditions (ULS) (T1375I)

Code	Unbonded				Bonded			
	Point A	Point B & C	Point D	Point E	Point A	Point B & C	Point D	Point E
ACI/IBC	2368	2631	1161	966	2463	2625	926	0
EC2	2915	3399(top) 1180(bot)	1134	0	2915	3399(top) 1180(bot)	1134	0
TR43	2703	3821(top) 1602(bot)	986	0	2836	3708(top) 1489(bot)	1134	0



Distribution of Shear (kN)

FIGURE 8.5-1

design shear (V_u) is computed from the results of the standard frame analysis performed for the loading conditions D, L and PT. The following combination was used:

The design starts with the calculation of vc, the code allowable shear stress contribution of concrete over the shear area of the section

$V_u = 1.2*V_D + 1.6*V_L + 1.0*V_{HYP}$
Span 1
b_w = 460 mm
d = 760 –40-22/2 =709 mm
point of zero shear
= 422.05*20/(422.05 + 555.87)= 8.63 m
Design at distance = column width/2 + d
For left support: 350/2 + 709 = 884 mm
For right support: 450/2 + 709 = 934 mm
For left support:
V_u = -422.05*(8.63-0.884)/8.63 = -378.82 kN
For right support:
V_u = 555.87*(20-8.63-0.934)/(20-8.63)
= 510.21 kN
Hence, the right support governs.
b_w = 460 mm
d = 0.8*h = 0.8*760 = 608 mm

Distance "d" can be calculated from the position of reinforcement in the section. However, ACI-318-11[43] stipulates that d need not be taken less than 0.8h. For hand calculation, this option is used conservatively.

d_p = 690mm > 0.8h = 608mm
Conservatively assumed 608mm
$v_{cmin} = 0.166*\sqrt{28}$ = 0.88 MPa
$v_{cmax} = 0.420\sqrt{28}$ = 2.22 MPa
v_c [44] $= 0.05*\sqrt{f'_c} + 4.8*V_u*d/M_u$
The term (V_u*d/M_u) must be less than 1 or use 1.
V_u*d/M_u = 510.21*608/(1412.22*1000)
= 0.22 < 1 OK
$v_c = 0.05*\sqrt{28} + 4.8*0.22$
= 1.32 MPa > $v_{c\,min}$ = 0.88 MPa
< $v_{c\,max}$ = 2.22 MPa
Hence v_c = 1.32 MPa governs the design

For this example the ultimate moment was taken at the face-of-support for brevity, while the shear check is done at a distance h/2 away from the support. This assumption is conservative and does not have a significant impact on the outcome of the calculation. In the general case, the value of V_u*d/M_u varies along the length of the member. But, it is assumed constant for this expeditious hand calculation.
v_u = (510.21*1000)/(460*608) = 1.82 MPa
> $\Phi\, v_c$ = 0.75*1.32 = 0.99 MPa
Hence shear reinforcement is required by calculation.
Assume 12 mm stirrups with two legs:
A_v = 2*129 = 258 mm²
The spacing, s, between the stirrups is given by:
$s = \Phi*A_v*f_v/[b_w(v_u - \Phi*v_c)]$
= 0.75*258*460/[460*(1.82 – 0.99)]
= 233 mm
also s<= 0.75*h = 0.75*760 = 570 mm
and s <= 600mm[45]

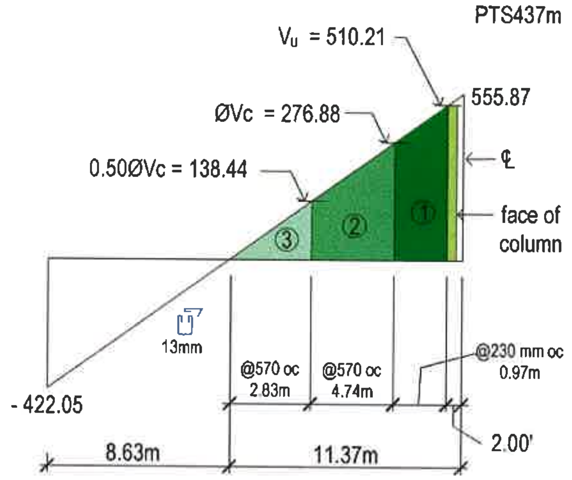
[43] ACI 318-11, Section 11.3.1
[44] ACI 318-11, Section 11.3.2
[45] ACI 318-11,Section 11.4.5.1

Select s = 230mm for the entire region where stirrups by calculation governs.
Using similar triangles, the three regions for the calculation of shear reinforcement are worked out and shown graphically in Fig. 8.5-2.
For the first region $V_u >= \Phi^*V_c$ = 460*608*0.99/1000 = 276.88 kN
Use stirrups at 230mm spacing.
For the second region $V_u >= 0.5^*\Phi^*V_c$ = 0.5*460*608*0.99/1000 = 138.44 kN
Use the minimum value specified by code.
For the third region $V_u < 0.5^*\Phi^*V_c$ = 138.44 kN
No web shear reinforcement required by code. Conservatively, use the same stirrups at 570 mm spacing (s <= 0.75 h = 570 mm).
For the region governed by the minimum rebar, the spacing shall be the smallest of the following:

In the following the three applicable code relationships [46] are rearranged to express them in terms of "s" spacing. The format of the relationships in the code is in terms of "A_{min}." In this case, since we have already selected a two-legged 12mm bar, we work out the spacing that is appropriate for our selection. Hence, $A_{min} = A_v$ = 2*129 = 258 mm^2.

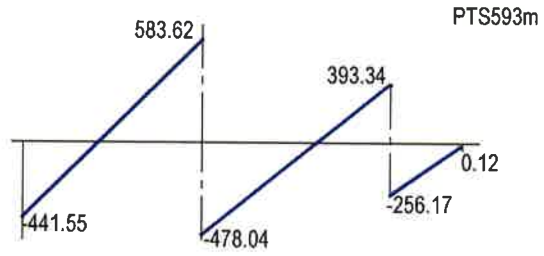
(i) $s = A_v f_y/(0.35\ b_w)$
= 258*460/(0.33*460) = 782 mm
(ii) $s = 80\ ^*A_v^*(f_y/f_{pu})^*d^*(b_w/d)^{0.5}/A_{ps}$
= 80*258 *(460/1860)*608*(460/608)$^{0.5}$/1188
= 2272 mm
(iii) $s = 16^*A_v^*f_y/(b_w^*f_c{}^{0.5})$
= 16*258*460/(460*280.5) = 780 mm
At the same time, spacing "s" shall not be more than 600 mm, nor 0.75*h=570 mm.
Use 12 mm two-legged stirrups at 570 mm on spacing for this region.

B. Based on EC2
$V_{ED} = 1.35^*V_D + 1.5^*V_L + 1.0^*V_{HYP}$
Span 1
b_w = 460 mm
d = 760 –40-22/2 =709 mm
Point of zero shear = 441.55*20/(441.55 + 583.62)
= 8.61 m
Design at distance = column width/2 + d
For left support: 350/2 + 709 = 884 mm
For right support: 450/2 + 709 =934 mm
For the left support:
V_{ED} = -441.55*(8.61-0.884)/8.61= -396.22 kN
For the right support:



Shear Force and Reinforcement for Right Side of Span 1 (kN; mm UNO)

FIGURE 8.5-2



Distribution of Shear (kN)

FIGURE 8.5-3

V_{ED} = 583.62*(20-8.61-0.934)/(20-8.61)
= 535.76 kN
Hence, the right support governs.
$V_{Rd,c}$ [47] = $[\ C_{Rd,c}{}^*\ k^*(100^*\rho 1^*\ f_{ck}\ ^*)^{1/3} + k\ 1^*\ \sigma_{cp}\ ]^*b_w\ ^*d$
but not less than $(v_{min} + k_1{}^*\ \sigma_{cp})\ b_w\ ^*d$

Where,
f_{ck} = 28 MPa
$k = 1+ (200/d)^{1/2} = 1+(200/709)^{1/2}$ = 1.53 < 2.0
$\rho 1 = Asl/\ (b_w\ d)$ = 9*.387/(460* 709) = 0.01068
$\sigma_{cp} = N_{ED}/A_C$ = 1428*10^3/917100 = 1.56 MPa
< 0.2*19 = 3.8 MPa
$C_{Rd,c} = 0.18/\gamma_c$ = 0.18/1.50 = 0.12
k_1 = 0.15
$v_{min} = 0.035^*\ k^{3/2*}\ f_{ck}{}^{1/2} = 0.035^*\ 1.53^{3/2*}\ 28^{1/2}$
= 0.35 MPa
$V_{Rd,c} = [\ 0.12^*\ 1.53^*(100^*0.01068^*\ 28^*)^{1/3} +$
$0.15^*\ 1.56\ ]^*460\ ^*709/1000$ = 262.18 kN

46 ACI 318-11,Sections 11.5.5.3 and 11.5.5.4

47 EN 1992-1-1:2004 (E) Section 6.2.2

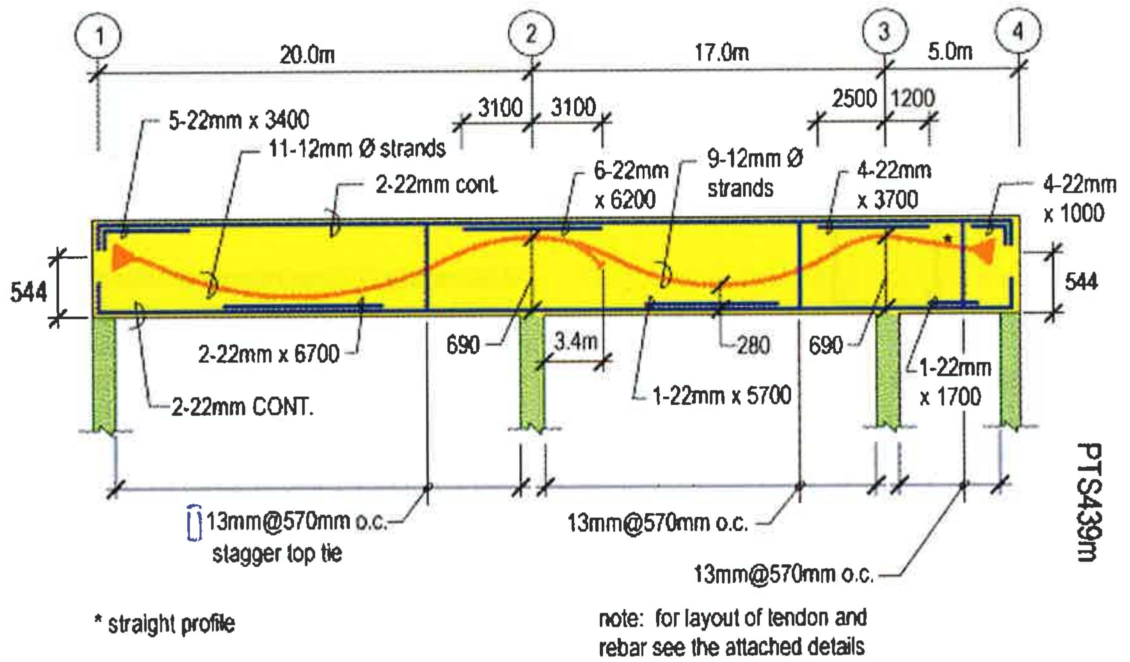
$V_{Rd,cmin}$ = (0.35 + 0.15* 1.56) 460 *709/1000
= 190.47 kN
$V_{Rd,c}$ = 262.18 kN
$V_{ED} > V_{Rd,c}$, Shear reinforcement is required by calculation.
Assume 12 mm stirrups with two legs
A_{sw} = 2*129 mm^2 = 258 mm^2
The spacing [48], s, between the stirrups is given by:
$s = (A_{sw}/V_{Rd,s})^*\ z^*f_{ywd}\ \cot\theta$
Where,
Assume $\theta = 40^0$, $\cot\theta$ = 1.20
$V_{Rd,s} = V_{ED} - V_{Rd,c}$ = 535.76– 262.18 = 273.58 kN
z = 0.9 d = 0.9*709 = 638 mm
s = [258 /(273.58*1000)]* 638*(460/1.15)*1.20
= 289 mm
$V_{Rd,max}$ [49] = $\alpha_{cw}{}^*b_w{}^*\ z^*\ \nu_1{}^*fcd\ /(\cot\theta + \tan\theta)$
Where,
ν_1 [50] = $0.6[1-(f_{ck}/250)]$ = 0.53 since $f_{ywd} > 0.8f_{yk}$
f_{cd} = 19MPa
α_{cw} [51] = $(1+\ \sigma_{cp}/f_{cd})$ for σ_{cp} = 1.56MPa < $0.25f_{cd}$ = 0.25*19 =4.75 MPa
α_{cw} = (1+ 1.56/19) = 1.08 α_{cw} equation is as per σ_{cp} values,

$V_{Rd,max}$ [52] = 1.08 *460* 638* 0.53*19 /(1.20 +0.84)
= 1564.59 kN > 273.58 kN OK
Select s = 280 mm
s [53] <= 0.75 d (1+cotα)= 0.75* 709 = 532 mm for the entire region where stirrups by calculation governs.
If $V_{ED} < V_{Rd,c}$, Use the minimum rebar specified by code [54].
For the region governed by the minimum rebar, the spacing shall be the following:

In the following the applicable code relationship is re-arranged to express it in terms of "s" spacing. The

format of the relationships in the code is in terms of "A_{min}." In this case, since we have already selected a two-legged 12mm bar stirrups, we work out the spacing that is appropriate for our selection. Hence, $A_{min} = A_v$ = 2*129 = 258 mm^2.

$s = A_{sw}{}^*f_{yk}/(0.08^*\sqrt{f_{ck}}\ ^*bw)$
= 258* 460 /(0.08*√28 *460) = 609 mm
At the same time, spacing "s" shall not be more than 532 mm.



Beam Elevation

FIGURE 10.1

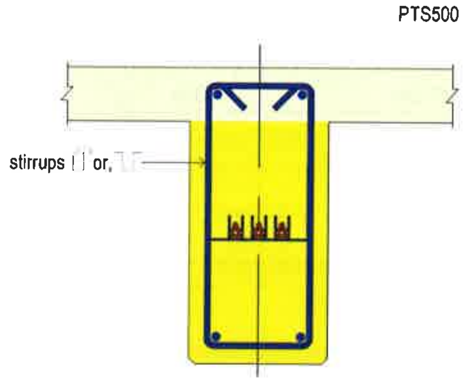
48 EN 1992-1-1:2004 (E) Exp: 6.8
49 EN 1992-1-1:2004 (E) Exp: 6.9
50 EN 1992-1-1:2004 (E) Exp: 6.6N
51 EN 1992-1-1:2004 (E) Exp: 6.11aN

52 EN 1992-1-1:2004 (E) Exp: 6.9
53 EN 1992-1-1:2004 (E) Exp: 9.6(N)
54 EN 1992-1-1:2004 (E) Exp: 9.4 &9.5(N)

TABLE 9-1 Stresses at Transfer of Post-Tensioning (T139SI)

	Point A	Point B	Point C	Point D	Point E
M_D (kN-m)	636.00	-926.00	-802.80	262.40	-68.88
M_{PT}(kN-m)	-434.80	592.70	495.30	-118.30	54.75
P (kN)	1428	1428	1428	1071	1071
1.15*P/A (MPa)	-1.79	-1.79	-1.79	-1.34	-1.34
f_t (MPa)	-2.85	0.12	0.03	-2.33	-1.27
f_b (MPa)	0.40	-5.72	-5.54	0.69	-1.45
ACI-08/IBC 2012					
F_t (MPa)	-12.00	1.12	1.12	-12.00	-12.00
F_b (MPa)	1.12	-12.00	-12.00	1.12	-12.00
	OK	OK	OK	OK	OK
EC2					
F_t (MPa)	-12.00	2.21	2.21	-12.00	-12.00
F_b (MPa)	2.21	-12.00	-12.00	2.21	-12.00
	OK	OK	OK	OK	OK
TR-43					
F_t (MPa)	-10.00	1.59	1.59	-10.00	-10.00
F_b (MPa)	1.59	-10.00	-10.00	1.59	-10.00
	OK	OK	OK	OK	OK

Note: Section properties I, A, S_{top}, S_{bot} are the same as used for service condition stress check
F_t and F_b are allowable stresses at top and bottom respectively.



(a) Arrangement of unbonded tendons

(b) Tendons support chairs

Placement of Tendons in Beam

FIGURE 10-2

Use 12 mm two-legged stirrups at 530 mm on spacing for this region.

C. Based on TR-43 [55]

TR-43 refers to EC2 for one-way shear design. But TR-43 includes the safety factor, γ_p, in the calculation of σ_{cp}. Where γ_p equals 0.9 if the prestress effect is favorable and 1.1 when it is unfavorable.

Precompresssion (σ_{cp}) enhances the shear capacity. Hence, in using the EC2 as outlined in the preceding, reduce the value of precompression by factor 0.9

9. CODE CHECK FOR INITIAL CONDITION

At stressing (i) concrete generally has not reached its design strength; (ii) prestressing force is at its highest value; and (iii) live load generally envisaged to be counteracted by prestressing is absent. As a result, the stresses experienced by a member can fall outside the envelope of the limits envisaged for the in-service condition. Hence, post-tensioned members are checked for both tension and compression stresses at transfer of prestressing. Where computed compression stresses exceed the allowable values, stressing is delayed until either concrete gains adequate strength or the member is loaded. Where computed tension stresses are excessive, ACI/IBC[56] suggest adding non-prestressed reinforcement to control cracking.

9.1 Load Combinations

The codes covered are not specific on the applicable load combination at transfer of prestressing. The following is the combination generally assumed among practicing engineers.

[55] TR-43 Second Edition, Section 5.9.1 and 5.9.2.

[56] ACI 318-11; Section 18.4

Load Case: 1.0*DL + 1.15*PT

Specification of this design example calls for tendons to be stressed with concrete cylinder strength is not less than 20 MPa.

f'_{ci} = 20 MPa

9.2 Stress Check

$\sigma = \pm(M_D + 1.15*M_{PT})/S + 1.15*P/A$

$S = I/Y_c$

9.3 Allowable Stresses

A. Based on ACI 318-11; IBC 2012

Tension = $0.25*\sqrt{20}$ = 1.12 MPa

Compression = 0.60*20 = -12 Mpa

B. Based on EC2

Tension = f_{cteff} = 2.21 MPa

Compression = 0.60*20 = -12 Mpa

C. Based on TR-43

Tension = $0.72f_{ctm}$ = 1.59 MPa

Compression = 0.50*20 = -10 Mpa

Farthest fiber stresses are calculated in a similar manner to service condition as outlined earlier. The outcome is summarized in Table 9-1.

If in any of the above locations the stresses exceeded the allowable values the following would have been done.

If compression stresses exceed the allowable value, the design parameters must be modified to bring the stresses within the code limits. If tensile stresses exceed the allowable value, bonded additional reinforcement (nonprestressed, or prestressed) shall be provided in the tensile zone to resist the total tension force in concrete computed with the assumption of an uncracked section.

10. DETAILING

The final tendon and reinforcement layout for the designed beam frame are shown in Fig. 10-1 through 10-3 for unbonded tendons.

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TR-43, *"Post-Tensioned Concrete Floors Design Handbook-Second Edition"*, The Concrete Society 2005.

ADAPT-PT (2012), *"Software for Design of Floor Systems and Beam Frames"*, ADAPT Corporation, Redwood City, CA, www.adaptsoft.com, 2012.

FIGURE 7.SI.1 Beam and Slab Construction; Beam Released from Column;
Tendons crossed and stressed on stem sides
(California; Courtesy DES and ADAPT; P731)

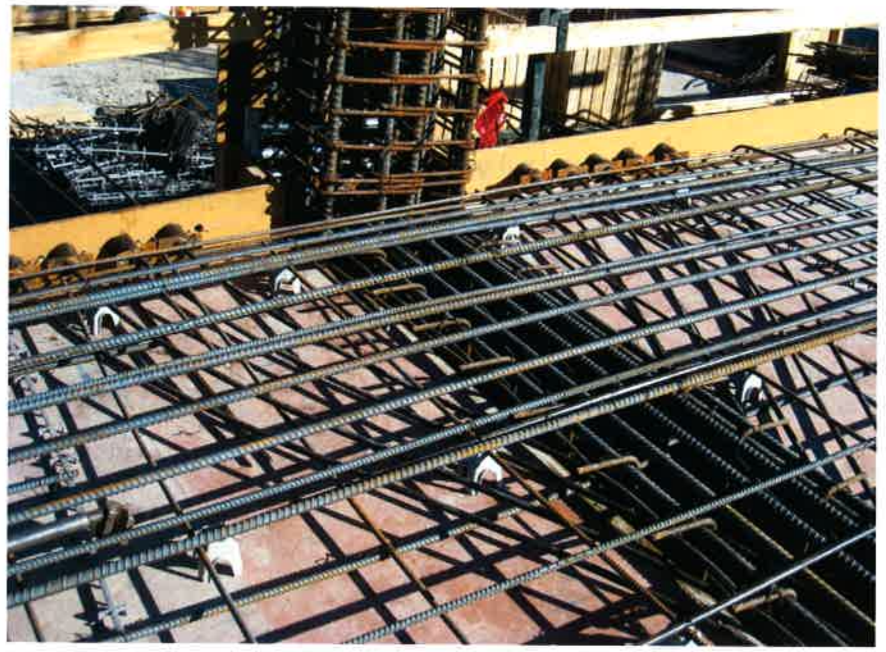


FIGURE 7.SI.2 Beam and Slab Construction;
Monolithic Beam-Column Connection;
A number of tendons are crossed over within the beam stem and raised to anchor at slab edge (California; Courtesy DES and ADAPT; P732)

CHAPTER 8

COMPUTER APPLICATION TO DESIGN OF CONVENTIONALLY REINFORCED, OR POST-TENSIONED BUILDINGS

Post-Tensioned Building in San Francisco
(Foundry Sq; courtesy Nishkian Menninger; P520)

8.1 OVERVIEW

The development and accelerated adoption of Building Information Technology BIM targets a seamless flow of information from the inception of a project to the completion of its construction. A critical link along the flow of information and inter-operability among the professionals is the structural engineers' work—a link that so far has not lent itself to be a trouble free part of the seamless chain. Meeting the challenge, new work has re-defined the way concrete buildings can be designed, however. Novel concepts for modeling, analysis and design of concrete buildings, within the realm of BIM environment, have now paved the way for a smooth flow of information from the architectural three dimensional building models, or drawings, to structural analysis, structural design, and the generation of construction documents.

This Chapter explains the challenges in the integration of structural engineer's work in the otherwise integrated flow of BIM, and outlines the procedures developed to overcome the failings. The Chapter concludes with an example.

8.2 BIM; BUILDING MODELING AND STRUCTURAL DESIGN PROCESS

The practice in BIM-based construction has been for the structural engineer to receive a three dimensional model that includes the physical features of the building. The structural engineer concludes with the definition of the material properties critical to the in-service performance and safety of the structure, such as concrete strength, amount, position, and configuration of reinforcement, and details of post-tensioning tendons, if applicable. The work of the structural engineer supplements the information of the building model. Structural and fabrication drawings are then expected to be extracted from the general package. Figure 8.2-1 illustrates the view of a building model. Figure 8.2-2 shows a structural analysis outcome extracted from one of the floors of general model. Details of the process, highlighting the difficulties and the solutions are explained next.

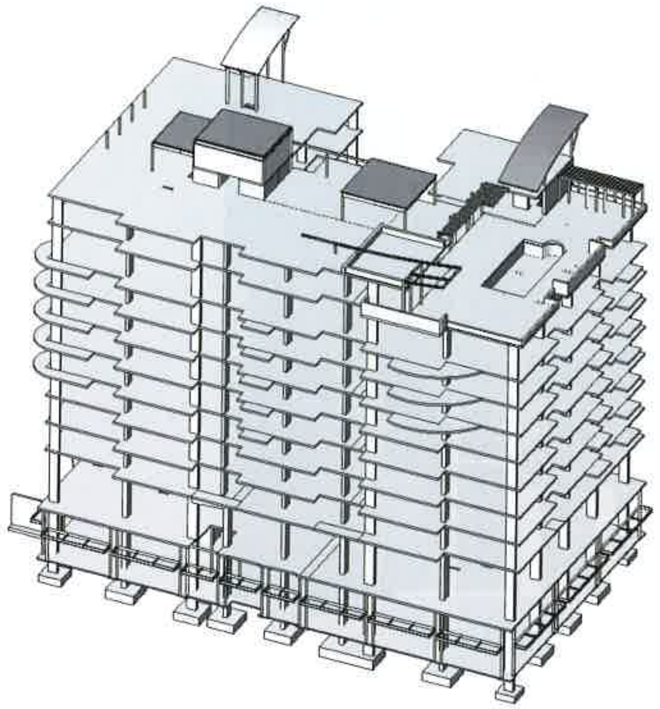


FIGURE 8.2-1 View of a Building Generated in 3D for BIM (P105)

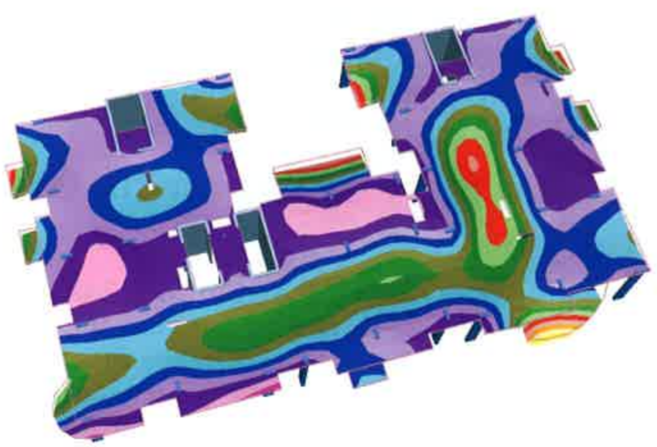


FIGURE 8.2-2 Floor System Extracted from 3D Model for Detailed Analysis (P106)

8.2.1 Physical and Analytical Models; Migration to Analytical Space

The traditional process of structural engineering design consists of the following steps: (i) start from a set of architectural drawings; (ii) idealize the geometry given in the architectural drawings to one that lends itself to structural analysis; (iii) envisage a load path that embraces the designated force-resisting members of the building—including member dimensions and connectivity that are intended for structural analysis, but may differ from the architectural drawings; (iv) perform structural analysis, using the geometry defined for analysis; (v) map the outcome of structural analysis to architectural drawings for detailing and construction. Figure 8.2.1-1 illustrates the process.

Figure 8.2.1-1a illustrates the "physical" model of a simple frame. It reflects the shape and dimensional features of the real structure. This information is reflected in the architect's drawings and the three-dimensional BIM model of the frame, if available. It is referred to as the "physical" model of the structure. The "physical" model contains the details of the actual construction—as we see it. In the traditional approach, using the physical model, the design engineer creates an "analytical" alternative, such as the one illustrated in parts (b) and (c) of the figure for structural analysis. The analytical model is generally based on the centroidal axes of the members' that the design engineer designates to resist forces. Note that in the analytical model (part c) the physical length of the column (H) is converted to the analytical length (h). The structural analysis generates design actions (moments, shears) for the analytical lengths. But the design engineer must deliver reinforcement drawings, and details, that are based on the physical model shown in part (d). Obviously, the information generated from the analysis, having been based on dimensions that differ from the architectural drawings, is not compatible with the general BIM models. The analysis results have to be mapped onto the dimensions of the physical model.

Ideally a design engineer would prefer to have the analysis and design based on the "physical" model, the backbone of the BIM part (f), and forego the process of switching between the two models. Developments in the analytical formulation of structural engineering have now made it possible to do without the "analytical" model, and perform an analysis and design based on the "physical" alternative, thereby enabling a seamless integration of structural engineer's work to BIM. The process is explained next:

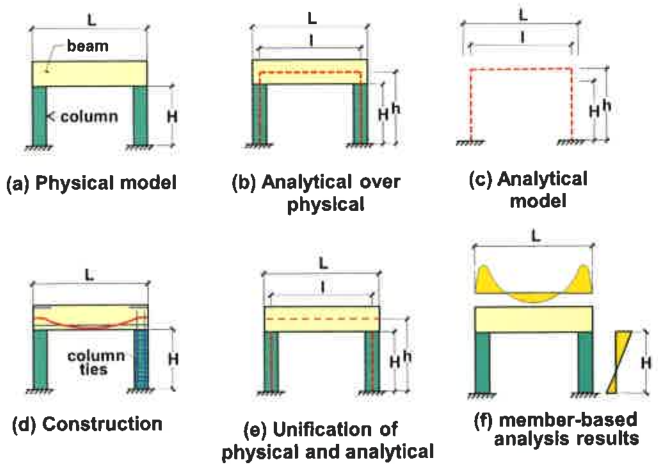


FIGURE 8.2.1-1 Physical and Analytical Models (P601)

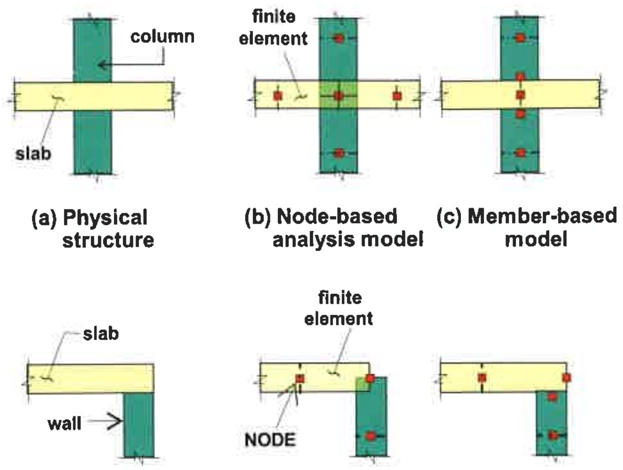
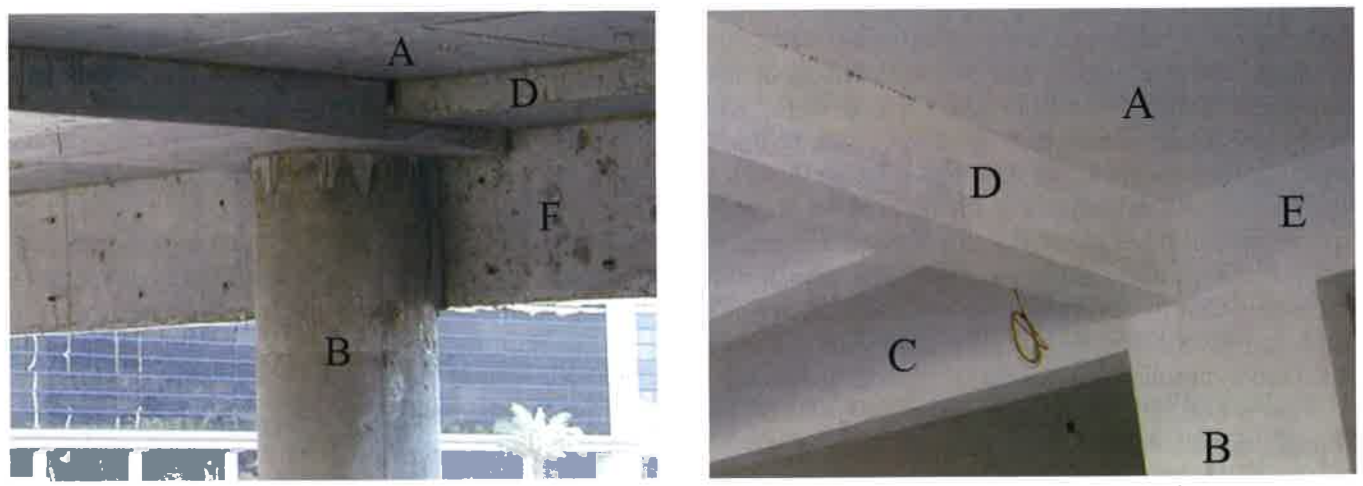


FIGURE 8.2.1A-1 Physical; Node and Member-Based Models (P600)

A. Node-Based Analytical Models: Figures 8.2.1A-1 (a) and (d) illustrate two common scenarios in building construction. Using finite element method (FEM), the slab/beam and column are represented by FEM elements. Each element enters the assemblage of the structural system matrix by way its "nodes," at which the handshake for force transfer from one element to the next takes place. As illustrated in parts (b) and (e) of the figure, the node of a column coincides with that of the slab for transfer of force from one to the other. The figures show overlap of the adjoining members. In part (b) the column length is extended into the slab and in part (e) the column and slab are modeled as in their physical prototype.

The node-based modeling scheme forms the basis of practically all FEM analysis tools currently available. The modeling scheme starts by defining a number of "nodes" that are typically along the centroidal axes of the physical members. Finite elements are created joining adjacent nodes. This requires intersection of the centroidal axes of the members that meet and share force.

In practice, and in particular in concrete construction, rarely a physical structure is conceived on the premise of the intersection of the centroidal axes of its members. Consider the typical concrete construction details shown in Fig. 8.2.1A-2. Evidently, the centroidal axes of the column and the multiple members identified in the figure do not converge, as required by the "nodal" based modeling. Using node-based modeling, the analysis requires special handling of the joint to capture and design the transfer of force from one member to an adjoining one, thus creating an analysis model.



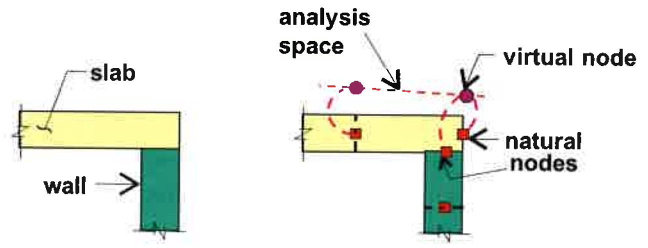
(a) Construction Detail (P107)

(b) Construction Detail (P108)

FIGURE 8.2.1A-2 Construction Details with Non-Intersecting Centroidal Axes of Members at the Joint

B. Member-Based Analytical Model—Virtual Analysis Space: In this process, the physical model is used in it's as-is geometry to analyze and detail the structure. This eliminates the generation of an "analytical" model that generally differs from the physical dimensions of the construction. The process becomes an integral part of BIM, enabling a seamless flow of information between the structural and other trades of the project.

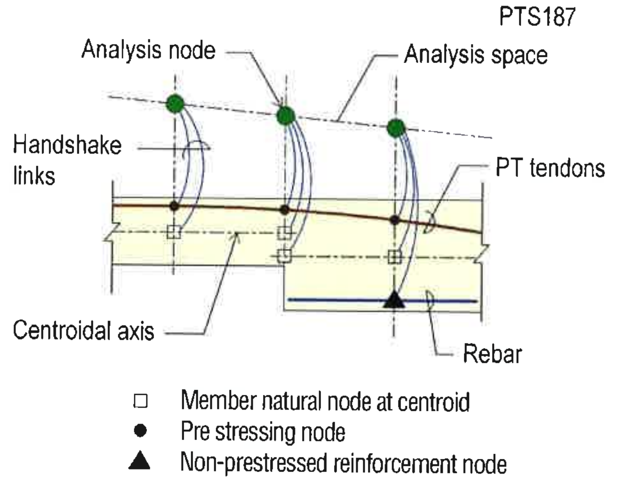
Consider Fig. 8.2.1A-1 parts (c) and (f). In each case, the column terminates at its interface with the slab. In part (f) the slab extends to the outer face of the column. In both cases, at the junction between the two members "slab" and "column", the slab is given "priority" to continue through the joint, and the column is terminated at its interface with the slab. This is based on the assumption that the design would favor the slab reinforcement to continue as shown, but the column ties terminate at the interface with the slab, and re-start above the slab, if column is continuous.



(a) Physical structure (b) Member-based model

FIGURE 8.2.1B-1 Virtual Nodes and Analysis Space (P602)

The process is achieved by departing from the traditional requirement of force transfer between one element and the next at the centroidal axes of the adjoining members—rather, the force transfer will be based on the intersection of member bounds. In this process, the finite element relationships for each structural member are formulated in an analytical space, instead of the traditional centroidal axis/plane of the members. The force transfer of members that intersect takes place at a virtual node that represents their common solid parts. Figure 8.2.1-1b symbolically shows the analytical space by a broken line. The structural response of each element is expressed in the analytical space, as opposed to the centroidal location of the physical members. The analysis takes place in the "analytical" space. It is then followed by transfer of deformations and ac-

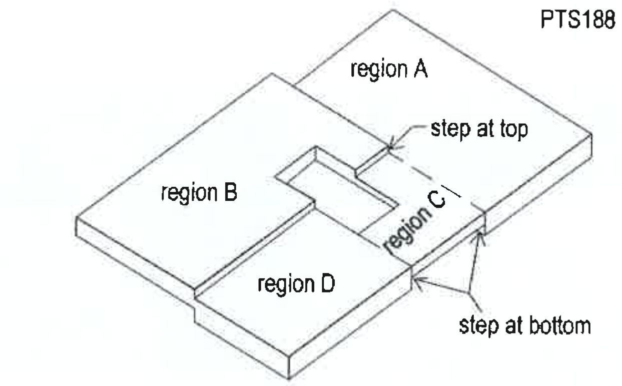


Presentation of Physical Member Objects and Mapping to Analysis Space

FIGURE 8.2.1B-2

tions back to the physical entities for section design, detailing and preparation of structural documents.

A side benefit of the physical-based formulation is that it enables the inclusion of rebar, as well as pre-tensioned and post-tensioned reinforcement in the analysis, as exemplified in Fig. 8.2.1B-2[1]. At the analysis node, each segment of rebar or post-tensioning steel contributes to the structure based on its physical properties and at its physical location in the structural member.



View of Slab Outline with Openings and Steps

FIGURE 8.2.1B-3

[1] Process implemented in ADAPT-ABI and ADAPT-Builder www.adaptsoft.com

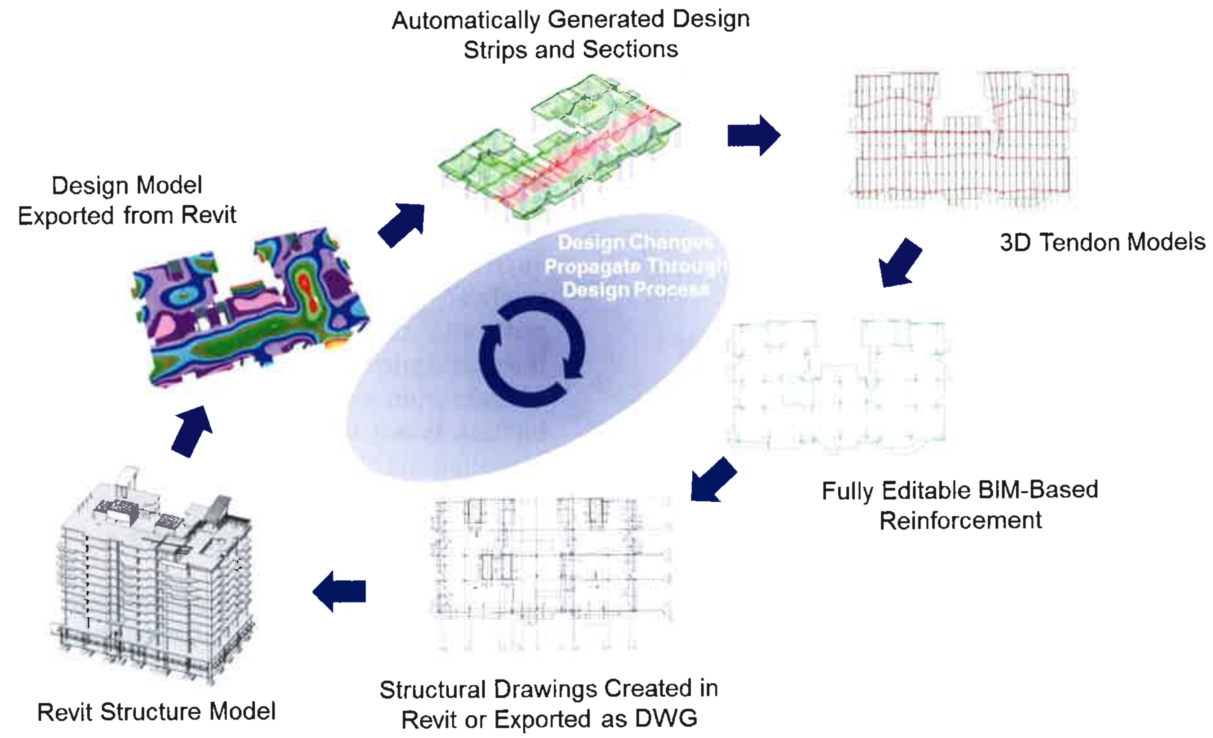


FIGURE 8.3-1 Example of Information Flow from a 3D Model to Structural Analysis and Return of Construction Information to 3D Model (P109)

The transformation of the structure to an analysis space enables the structural design of complex floor systems, such as the partial view shown in Fig. 8.2.1B-3 to be followed with the same ease as slabs of uniform thickness. Beams, steps, openings, and complex physical details can all be modeled as they appear in the prototype.

8.3 INTEGRATION OF STRUCTURAL ANALYSIS IN BIM

Introduction of analysis space provides the missing link for the seamless flow of information in the BIM process from the architectural conception to the construction documents. A brief practical example of the process is given next.

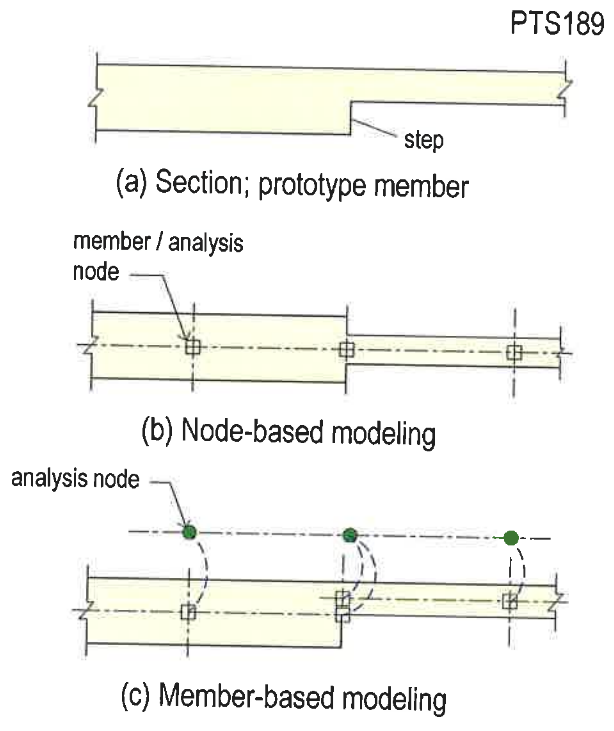
The floor level shown in Fig. 8.3-1 with color contour is extracted from a three dimensional model of the structure[2]. Using the true geometry of the structure the model is analyzed. Post-tensioning and base reinforcement, such as mesh is added, where applicable. Going through the structural design process, added reinforcement is determined, and fabrication drawings generated. The entire reinforcement is then passed on to the 3D model for further processing and to be shared among the various trades.

[2] Model generated using Revit Structure; structural analysis and generation of construction documents performed using ADAPT-Builder

8.4 APPROXIMATION IN ANALYSIS

For many engineers, a finite element method (FEM) based structural analysis software acts as a "black box," that delivers the design values they need, once the program is fed with the information it requires. Often, repeated unquestioned use of software creates the perception of its validity, or accuracy of its outcome, thus reinforcing reliance on its use. FEM-based programs do not all share the same formulation, nor do they deliver the same accuracy. There are differences in the simplifying assumptions made in modeling of a floor system, differences in internal formulation of software and the manner each extracts and reports the information required for design. This section highlights several of the underlying differences among the commercially available software.

A. Approximation in Modeling the Physical Structure and Connectivity: Apart from the node-based or member-based modeling techniques that are discussed in the preceding, most software go through additional simplifications for analysis. The simplifi-



Node and Member Modeling Presentations of Steps

FIGURE 8.4A-1

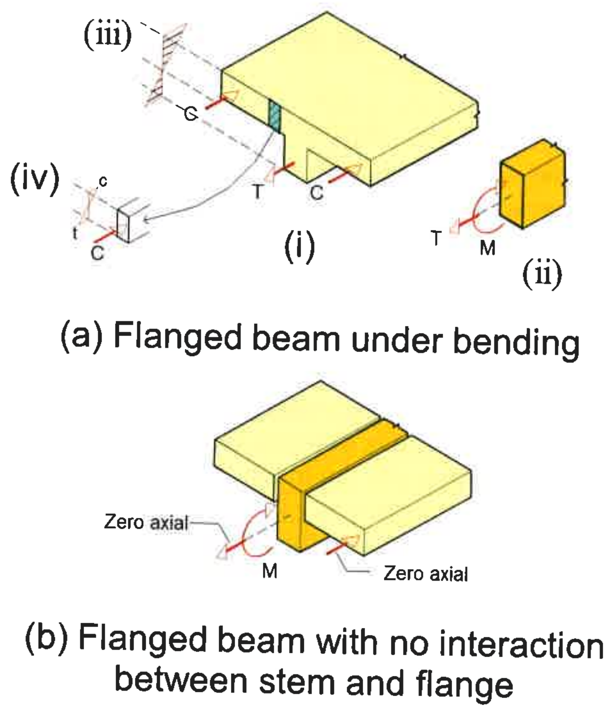


FIGURE 8.4A-2 Beam Modeling Options (P603)

cations generally take place in the background—not always apparent when viewing the model. The simplifications are mostly necessitated due to the application of "node-based" formulations.

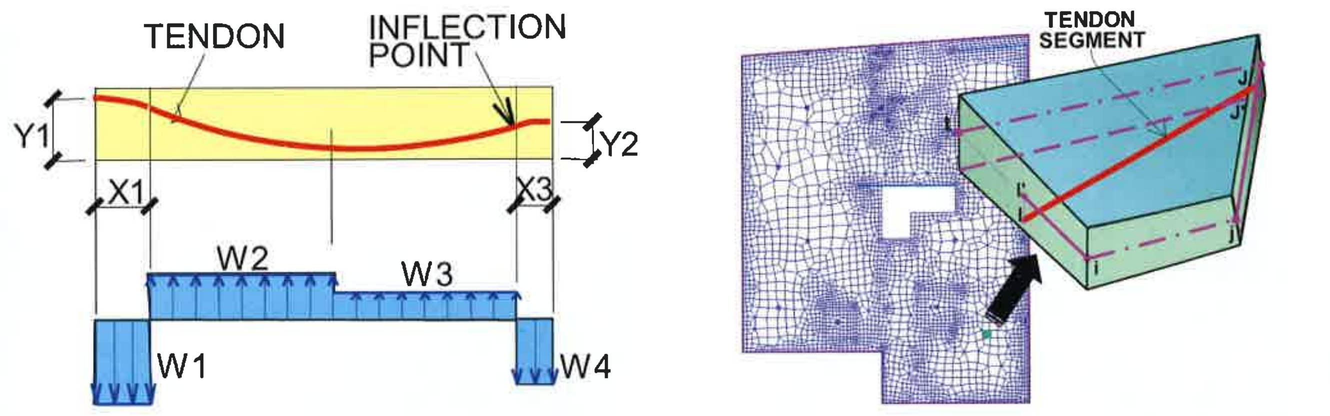
The most common simplification is modeling of changes in thickness, or change in elevation of a floor slab. Such a change, as illustrated in Fig. 8.4A-1 is mostly modeled as shown in part (b) of the figure, where the centroidal axes of the two parts are lined up to satisfy the prerequisite of intersection of centroids for force transfer from one slab region to the next. This approximation is not followed, when using member-based modeling coupled with "analysis space." As illustrated in part (c) of the figure, each slab region is positioned, where it occurs in the physical space.

Using node-based modeling, the stem of a flanged beam must be raised to align its centroid with that of the adjoining slab. Alternatively the beam stem must be considered disjointed from the flange as shown in Fig. 8.4A-2b. With member-based modeling, beam and slab construction enter the analysis as they occur in the physical state of the structure, and thus enabling correct interaction among the structural constituents of a floor.

In the node-based modeling, the value obtained from the analysis for the isolated beam is reported as "beam" value. In the member-based modeling, since the beam and slab act together to resist the applied forces, the beam stem generally includes axial force that is the outcome of the stem's interaction with the slab.

B. Approximations in Tendon and Rebar Modeling: Almost all commercial software used for design of floor systems, model pre-. or post-tensioning tendons through forces that each exerts to the structure as shown in Fig. 8.4B-1a. A prerequisite to this modeling is that the force in tendon must be known and specified prior to "analysis." Once a tendon is considered removed from the structure, its stiffness can no longer participate in the deformation of the structure, nor can the flexing of the structure be accounted for in changes of tendon stress.

Improved analysis procedure considers tendons as members resisting the applied forces, along with the concrete that contains them (Part b). The same scheme applies to the reinforcement that may be pre-defined by designers before an analysis. Modeling reinforcement in addition to pre-stressing steel is critical for a reliable estimate of a floor's cracking,



(a) Traditional representation of a tendon through equivalent forces (P604)

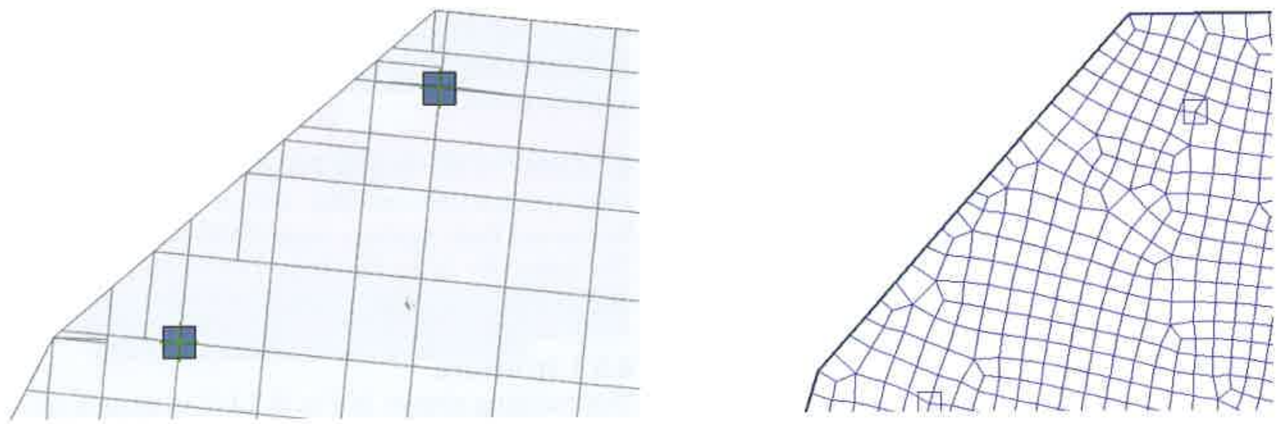
(b) Representation of tendons as discrete segments of pre-tensioned steel within each finite element (P605)

FIGURE 8.4B-1 Options in Representation of Prestressing Tendons

crack width estimate and cracked slab deflection, since the post-cracking response depends on the presence, location and amount of reinforcement.

C. FEM meshing: Meshing is the subdivision of a structure into finite elements for analysis. Most software use triangular, rectangular, or quadrilateral elements. A quadrilateral element generally provides a more accurate solution than the triangular option. A central requirement in the node-based subdivision of a member into finite elements is that the nodes of adjoining elements coincide with one another, in order to establish equilibrium of the externally applied forces with those generated in the structure. The equilibrium of forces is established at the nodes. Figure 8.4C-1 shows partial plan of two meshing options. Note that in part (a), at several instances the node of one cell falls on the edge of an adjoining cell, rather than coinciding with a node of the neighboring cell. Such conditions lead to an implicit gap between the affected cells. To close the gap, external forces, in addition to those defined by the designer are required to be added at the location. The forces introduced to close the gaps evidently impact the analysis results, and consequently the design values. It is not always apparent, whether the impact will result in a "conservative" or "un-conservative" design.

D. Creation of Design Strips: Following the procedure outlined in Chapter 3, at design stage a floor is subdivided into design strips. The design strips represent the designer's-designated load paths. A prerequisite of this process is that each location on a floor system be assigned to a design strip (load path), and be designed for the forces determined for that location.



(a) Meshing with cells not joining at common nodes (P111a)

(b) Mesh with cells joining at common nodes (P111b)

FIGURE 8.4B-1 Options in Representation of Prestressing Tendons

FIGURE 8.5.1-1 Building under Construction (P113)

FIGURE 8.5.1-2 Reinforcement Layout of a Typical Floor (P114)

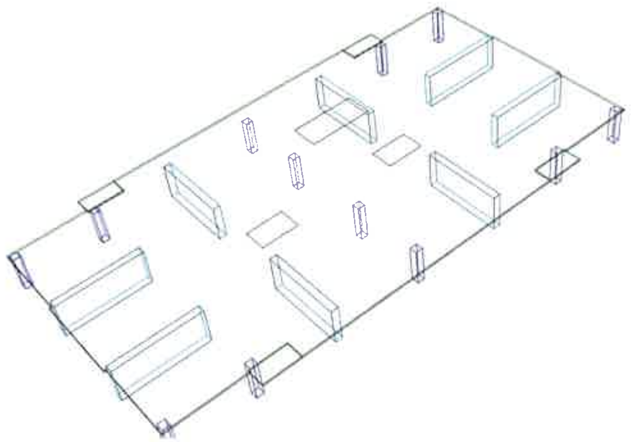


FIGURE 8.5.1-3 Analytical Model of the Floor System (P115)

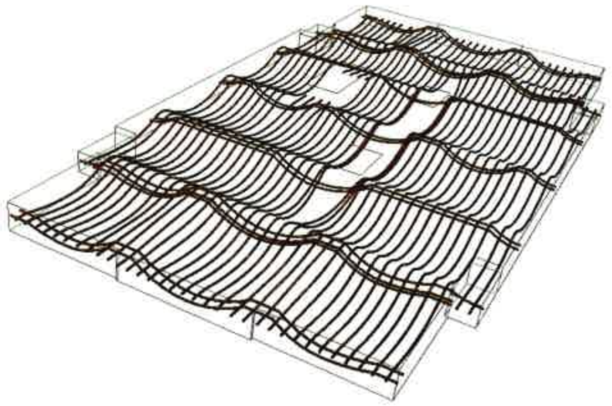


FIGURE 8.5.1-4 Arrangement of Tendons in the Floor System (Slab thickness is exaggerated for increased clarity) (P116)

E. Extraction of Design Values: There are several options to determine the "design values," such as design moment and shear, from the outcome of a FEM analysis. The extracted values can be based on the (i) integration of the stress values within the finite element cells, referred to as "stress integration"; (ii) summation of forces at the nodes, referred to as "nodal integration"; or (iii) "extended nodal integration," that is based on the nodal values and enables computation of design values of arbitrary length, location and orientation. From the same FEM analysis results, the design values extracted using the methods enumerated can be widely different. Description of the methods and their comparison is referred to in Chapter 4 and detailed in reference [ADAPT-TN 302, 2009]. The extended nodal integration results in values that are in equilibrium with the externally applied loads.

8.5 COMPUTER-BASED DESIGN EXAMPLE

This section presents the design of a simple post-tensioned floor system, using FEM-based software[3]. The objective is to illustrate the steps in modeling and design.

8.5.1 Structure

The building shown in Fig. 8.5.1-1 is located in San Francisco Bay Area—one of the highest seismic risk areas. The floor system of the building was de-

[3] ADAPT-Builder Floor Pro, www.adaptsoft.com

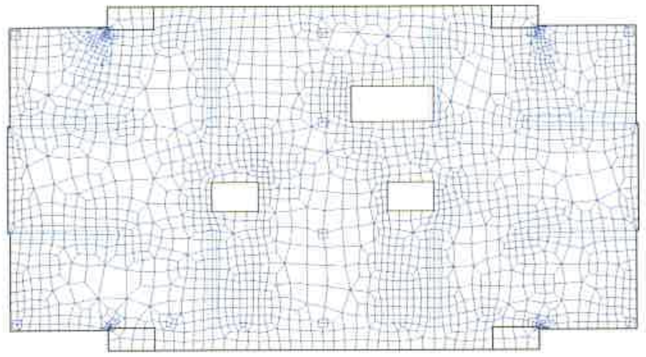
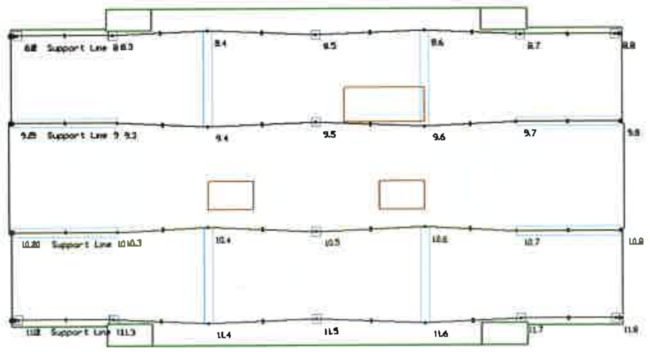
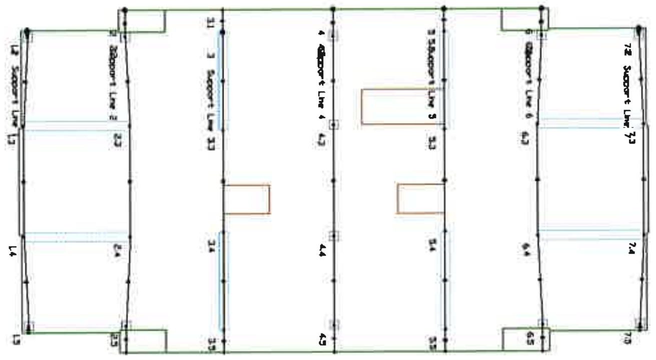


FIGURE 8.5.1-5 Discretization of the Floor System for Analysis Using an Adaptive Mesh Generator (P117)

FIGURE 8.5.1-6 Deflected Shape of the Floor System (P118)

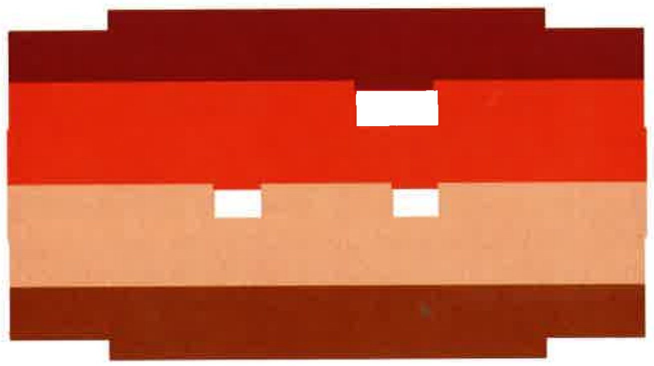


(a) Support lines in X-X direction (P119)

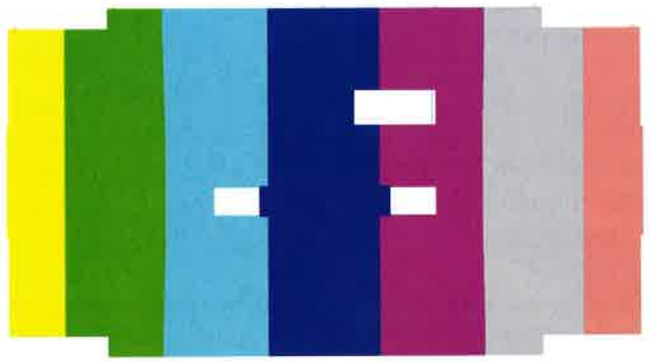


(b) Support lines in Y-Y direction (P120)

FIGURE 8.5.1-7 Support Lines in Two Orthogonal Directions

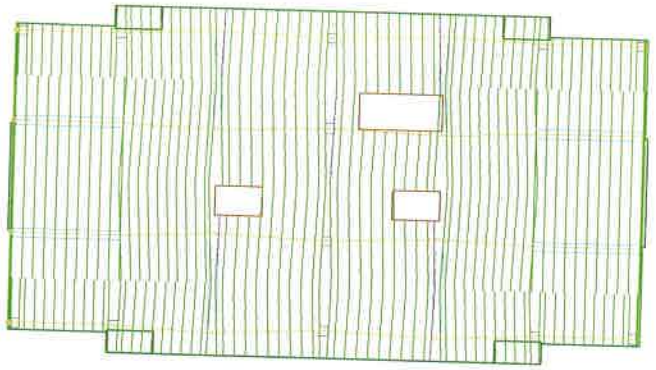


(a) Design strips along X-X direction (121)

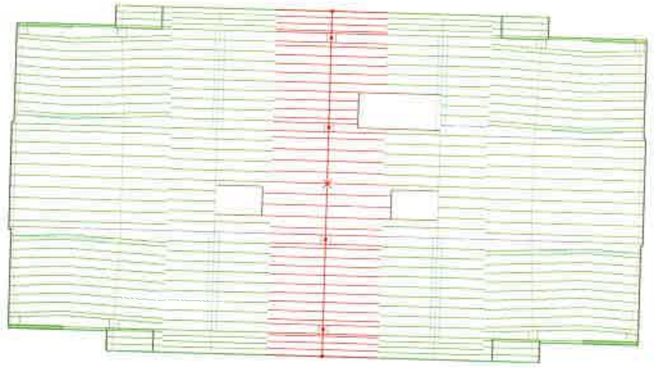


(b) Design strips along Y-Y direction (122)

FIGURE 8.5.1-8 Design Strips in Two Orthogonal Directions



(a) Design sections along X-X (P123)



(b) Design sections along Y-Y (P124)

FIGURE 8.5.1-9 Design Sections along the Two Major Axes

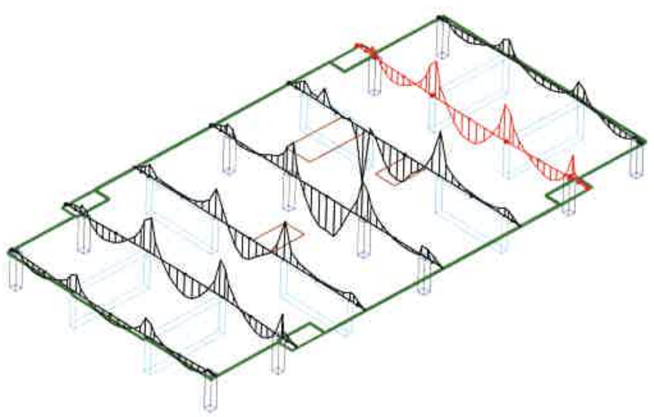


FIGURE 8.5.1-10 View of Design Moments for Design Strip along Y-Y (P125)

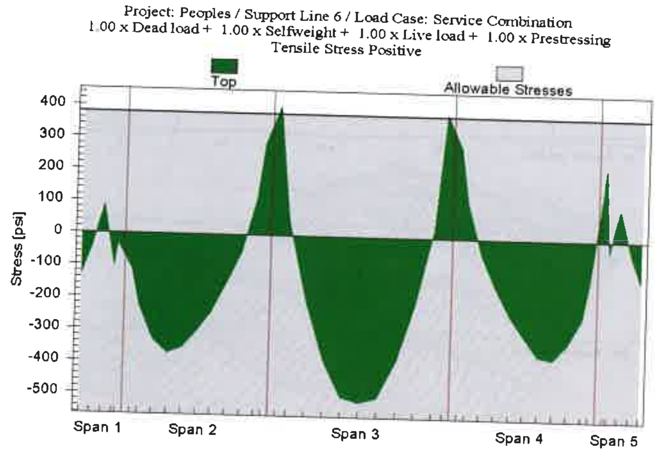


FIGURE 8.5.1-11 Distribution of Stresses at Top of Slab for a Selected Design Strip (P126)

signed using a flat slab construction reinforced with unbonded post-tensioning. Shear walls provide the lateral force resisting system of the building. Figure 8.5.1-2 shows the reinforcement layout of one of the typical levels.

The three dimensional computer model of the structure (Fig. 8.5.1-3) includes all the features of the floor system, such as steps at the balcony, openings and columns offset from the slab edge. The tendon layout is added to the model (Fig. 8.5.1-4).

Prior to analysis and design, along with tendon layout, engineers can specify base reinforcement in the structure. Next, the structure is meshed (Fig. 8.5.1-5) and analyzed. Figure 8.5.1-6 illustrates the deflected shape of the structure for one of the service load conditions. Next support lines are generated in two orthogonal directions (Fig. 8.5.1-7). This is followed by the generation of the associated design strips (Fig. 8.5.1-8).

Next, design sections are generated for each design span. The sections typically start at the face of the wall or column supports and are adequate in number to capture the distribution of design values for each span (Fig. 8.5.1-9).

The design values such as moments (shown in Fig. 8.5.1-10) are computed for each of the design strips and are used to determine the distribution of hypothetical stress (Fig. 8.5.1-11), and the associated reinforcement. The figure shows the computed stress against the background of allowable values, affording a rapid identification of locations where stresses exceed the allowable values. Sections that do not meet the code requirements are highlighted (Fig. 8.5.1-9) for ease of identification.

The integrated actions along each of the design strips are used for code compliance and determination of reinforcement. Figure 8.5.1-10 shows the distribution of design moments for design strips along Y-Y direction. The extreme fiber stresses associated with each of the design strips is used for stress check. Figure 8.5.1-11 shows the distribution of top stress (hypothetical values) against the background of allowable values.

The reinforcement for each location is determined and detailed on plan, ready for generation of the structural document.

8.6 REFERENCES

ADAPT- TN302, (2009), *"Evaluation of Actions at Design Sections Using Finite Elements,"* www. adaptsoft. com , pp. 9.

FIGURE 8.6-1 Grouted Slab (Morocco; P720)

FIGURE 8.6-2 Tendons Cross Over and are Stressed at the Sides of the Beam, Where Tendons Can Not Extend Through The Column (California; P730)

FIGURE 8.6-3 Unbonded Slab (UK; P722)

FIGURE 8.6-4 Construction Joint Unbonded Slab (US; P723)

FIGURE 8.6-5 Unbonded Slab (La Paz, Bolivia; P724)

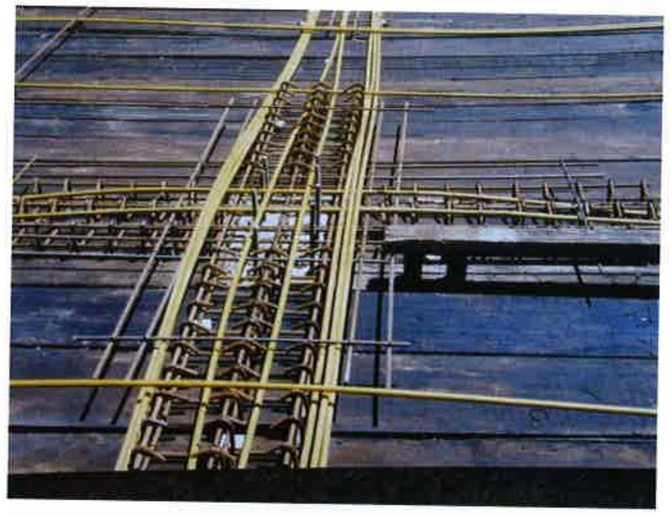


FIGURE 8.6-6 Punching Shear Reinforcement
(La Paz, Bolivia; P725)

FIGURE 8.6-7 Unfavorable Termination of Tendons All at the Same Location.
Stagger dead ends and provide bars to spread tendon force to front of anchors. (P726)

FIGURE 8.6-8 Delivery of Cut Strand to Job Site for Grouted Construction
(Beijing; P727)

FIGURE 8.6-9 Grouted Residential Foundation
(US; P728)

FIGURE 8.6-10 Unbonded Residential Foundation
(US; P729)

CHAPTER 9

POST-TENSIONING IN MULTISTORY BUILDINGS

Post-Tensioned Multi-Story Building; Escala, Seattle
(Engineered by Cary Kopczynski; photo by Michael Walmsley; P695)

9.1 STRUCTURAL IMPACT OF POST-TENSIONING IN MULTI-STORY BUILDINGS

The primary design considerations regarding the effects of post-tensioning in multi-story buildings, such as the one shown in Fig. 9.1-1 are: (i) changes in the axial forces and moments of the supporting columns and walls; and (ii) changes in the precompression of the floors due to constraint of supports to free shortening of the floors. There are other effects, such as creep and differential shortening of the supports that are common between a post-tensioned or conventionally reinforced building.

9.2 EFFECTS OF POST-TENSIONING ON COLUMN AND WALL SUPPORTS

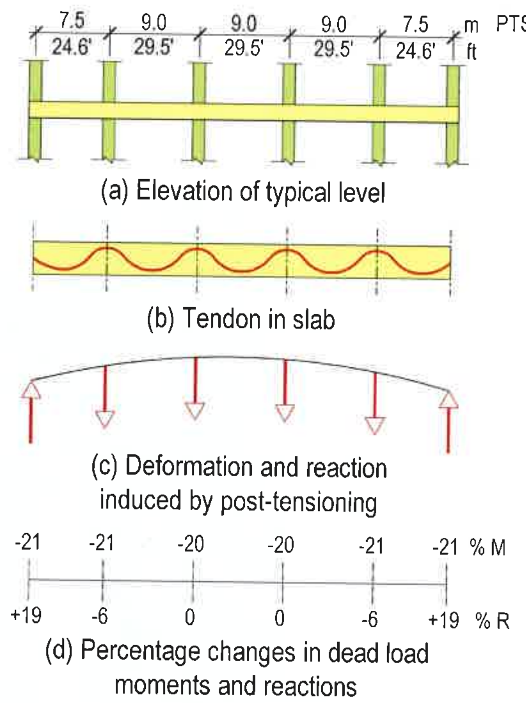
The common practice in design of multistory buildings is to post-tension the floors only. Columns and wall supports are generally constructed using conventionally reinforced concrete. To illustrate the impact of post-tensioning on a floor's supports, consider a typical floor having equal spans and a regular support layout as shown in Fig. 9.2-1. The floor geometry and support layout of most buildings are more complex, and the multi-level accumulated impact of post-tensioning on the supports will be less predictable. The example selected illustrates the concept, however. Detailed analysis maybe warranted for complex construction geometries.

Apart from forces in a floor, post-tensioning results in reactions in the column and wall supports. The reactions are due to the restraint of a floor's supports to free shortening and flexure of the floor, when the floor is post-tensioned. For purposes of illustration, it is convenient to consider the post-tensioning as a series of applied loads, referred to as "balanced" loads. The balanced loads from post-tensioning consist of forces normal to the plane of a floor, axial forces in plane of the floor, and moments at changes in the position of a floor's centroidal axis (see Chapter 4 for details). The reactions generated at the supports from the post-tensioning forces are referred to as "hyperstatic" forces. The hyperstatic actions from post-tensioning directly impact the strength design of the walls and columns. Hufnagel and Kang have investigated the secondary effects of post-tensioning on column reactions and reported their work in reference [Hufnagel et al, 2012].

Figure 9.2-1a illustrates a typical level of a high-rise building, supported on columns. The columns are on a 9 m (29.5') orthogonal grid. Part (b) of the figure shows the post-tensioning layout. At stressing of tendons the upper columns/walls are not in place. Or, if they are, they do not provide a design-significant restraint, since their upper ends would be free. Hence, the reactions from prestressing will develop only at the bottom supports.

To illustrate the phenomenon, let us assume that the post-tensioning applied balances 100% of the selfweight of the floor system. In most designs a smaller percentage of the selfweight load is balanced. We will next determine to what extent the selfweight reactions on the columns will be affected by the post-tensioning. Next, we will express the changes in selfweight reactions from prestressing.

Figure 9.2-1c illustrates the deformation tendency of the floor system from the post-tensioning, if the floor were free to deform. Presence of supports results in



Impact of Post-Tensioning on Typical Level of a High Rise Building

FIGURE 9.2 1

compressive forces at the far end supports and tension at the interior supports. The sum of the axial reactions on the supports from prestressing (part c) is zero, however. In effect, the prestressing results in a re-distribution of axial forces in a floor's supports.

The percentage change in selfweight reactions resulting from the application of post-tensioning is shown in part (d) of the figure. The maximum impact on the reactions (R) is at the outermost columns. Observe that the reaction from the selfweight load on the outermost column will be increased by 19% of its value. However, the reaction on the penultimate support will be decreased by 6%. The remaining reactions do not change to a design- significant extent. The maximum change in design reaction is at the supports of the end spans. The values shown are derived from the single level shown in part (a) of the figure for a uniformly loaded concrete frame having equal spans. However, they apply to any level within a high-rise, assuming that the levels are similar. Both the selfweight and post-tensioning reactions are accumulative over the stacked levels.

Comment
In the above example, it was assumed that the supports extend only one level below the floor. In the upper levels of multistory buildings, the longer length of a column from the level under consideration to the foundation results in distribution of reactions different from the one shown above. Column supports of upper levels are more flexible on account of their longer length from the foundation. The increase in flexibility of columns supporting higher levels due to increase in column length from the foundation results in different distribution of moments and reactions among different floors having the same floor geometry and gravity loads.

The change in selfweight moments at the supports due to prestressing is shown above the line in part (d) of the figure. Post-tensioning reduces the moments from selfweight at each support by the percentages indicated. The maximum reduction in moment is 21% for the end supports. The moment reduction for the interior columns is not of design-significance. As a matter of fact, column moments for uniform loads and spans become negligible at interior supports. The critical impact remains at the end and penultimate supports, and where spans and loads are non-uniform.

The following conclusions can be drawn for a generally regular structure. For irregular layout and loads, and where the supports are restrained at their far ends, specific analysis needs to be carried out.

- For uniformly loaded and regular concrete frames, the impact of post-tensioning results in an increase in the axial force at the end supports; reduction of the axial forces at the penultimate supports, and design-insignificant impact on the axial forces of the remainder supports.
- Post-tensioning reduces the design moments for "strength condition" at the top of member supports.
- Post-tensioning in a floor results in redistribution of axial force on walls and columns. However the sum of the axial forces for any give floor remains unchanged.

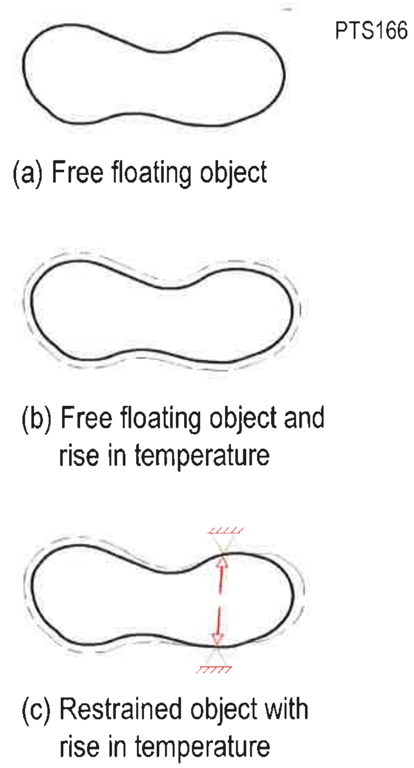
9.3 PRECOMPRESSION FROM POST-TENSIONING AND RESTRAINT OF SUPPORTS

Precompression results in shortening of a floor, somewhat analogous to the effects of a temperature drop. It is beneficial to briefly review the structural impact of temperature change in multi-story buildings, before highlighting the differences between the two effects.

Guo et al, [Guo, et al, 2013] carried out a numerical example of a multistory building to determine the effect of story-by-story post-tensioning on multistory buildings.

9.3.1 Temperature Effects

Consider a homogeneous object such as shown in Fig. 9.3.1-1a. A uniform change in temperature through the entire object is accompanied by an overall expansion, or contraction of all parts of the object. If the object is free, it will deform unimpeded (part b). No internal forces will be generated in the object. However, if the free expansion or contraction of the object is restrained (part c), the restraint to free deformation generates internal forces in the object. The internal forces generated due to the restraints are larger at the point of restraint and drop in value with distance from the restraint.



Free Floating and Restrained Objects and Temperature Rise

FIGURE 9.3.1-1

Figure 9.3.1-2a illustrates the elevation of a multistory frame, with fixed connections to the foundation. Since the connections of the frame to the foun-

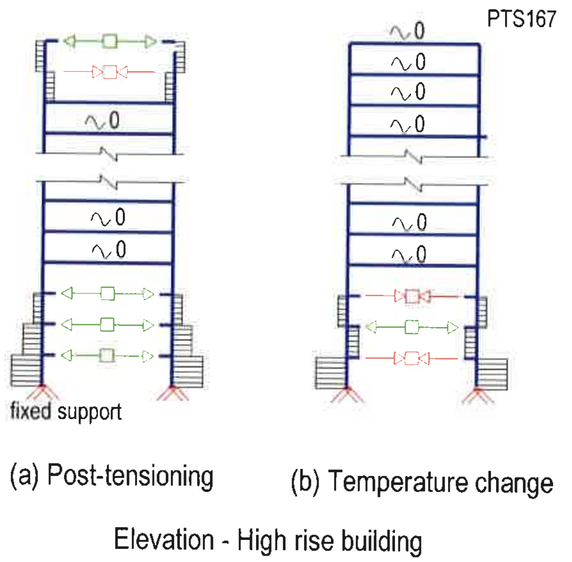


FIGURE 9.3.1-2 Distribution of Axial Force in Slab and Shear in Vertical Members from Post-Tensioning and Change in Temperature

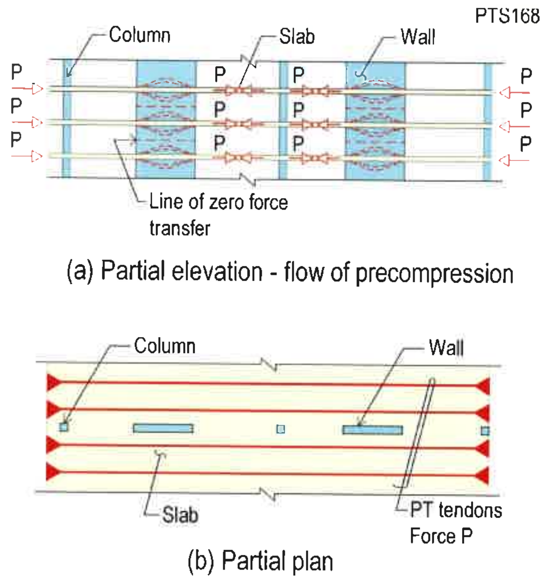


FIGURE 9.3.2-1 Precompression from Post-Tensioning at Upper Levels of Multistory Building

dation are fixed, restraining forces will be generated at these connections. Drop in temperature results in tension in the first elevated floor, a much smaller compression in the second floor, with rapidly decreasing impact at upper levels (Fig. 9.3.1-2b). In typical structures, the impact of uniform temperature change on the third level and beyond is not of design-significance.

9.3.2 Precompression from Prestressing
The primary difference between the interstory distribution of precompression in a typical floor, and temperature stresses arises from the fact that in multistory buildings, floors are constructed and stressed sequentially. Temperature stresses discussed above were applied to a completed building frame, whereas prestressing is applied contemporaneous with progress in construction.

Consider Fig. 9.3.1-2b. At time of application of prestressing to the first elevated floor, the second floor is generally not present. The restraint of the supports and foundation absorb a fraction of the precompression intended for the floor being stressed. When the second floor is post-tensioned, the restraint of its supports is somewhat less than experienced by the floor below it. Again, a fraction of the precompression of the new floor is diverted to the structure below it. This results in partial recovery of loss of prestressing in the first floor.

With progress in construction, at a typical upper level, the difference in the shortening of a floor being stressed and the floor immediately below it is the shortening that has taken place in the level below during the construction cycle of a floor. For typical buildings, the construction cycle is about 7 days. There will be an initial loss of precompression to the

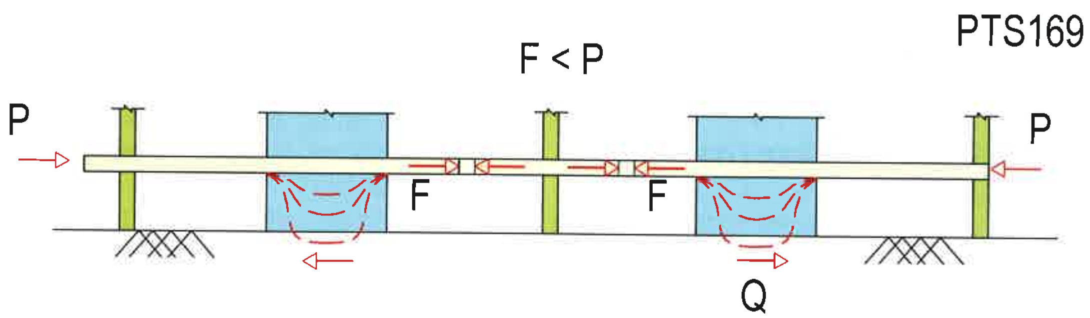


FIGURE 9.3.2-2 Diversion of Post-Tensioning From First Elevated Floor

level below, but the loss will be recovered when the level above is constructed and stressed. The pattern will continue, until the uppermost level is stressed. The precompression lost to the penultimate floor from the uppermost is not recovered, since there is no floor above it.

With lapse of time, over a period of two years or more, shortening due to creep, shrinkage, relaxation in prestressing and the effect of aging in concrete between a typical upper level and one above or below it becomes indistinguishable. This equilibrating final shortening of upper levels leads to a gradual readjustment of precompression among the upper levels, resulting in no additional transfer of precompression from one upper level to the other. Each typical level will then be acted upon by precompression from its own post-tensioning. This is valid except for the lowest levels, where the loss of precompression to the foundation will change with lapse of time, but will not be eliminated, as is the case for upper levels.

It is important to note that the presence of interior walls, or elevator core walls do not invalidate the conclusions arrived at in the preceding. Figure 9.3.2-1 shows a typical design strip of an upper level floor with elevation of two adjacent levels. Note that, subsequent to the long-term effects of creep and shrinkage, the precompression from each level diffuses into half the height of each wall immediately above and below the floor, but is fully recovered passed the end of a wall. Walls do not act in transferring precompression forces from one level to the next. This is based on the assumption that the geometry and prestressing of the adjacent levels are essentially the same.

The conclusion of no restraining effects from the walls does not apply to the first elevated floor over foundation, where the foundation does not move over time. The precompression lost to the foundation level is not likely to be recovered in full (Fig. 9.3.2-2).

9.4 REFERENCES

Guo, G., Joseph, L. M., and Darwin, D., (2013), *"Effects of Story-by-Story Post-Tensioning on Multi-Story Buildings,"* ACI Structural Journal, July-August 2013, pp. 649-657.

Hufnagel, A., and Kang, T.H., (2012), *"Assessment of Secondary Effects in Post-Tensioned Flat Plates,"* PTI Journal, Dec. 2012, PP. 26-42.

FIGURE 9.4-1 Construction Joint with Intermediate Stressing (Italy; P730)

FIGURE 9.4-2 Stressing Pans at Top of Slab (Italy; P731)

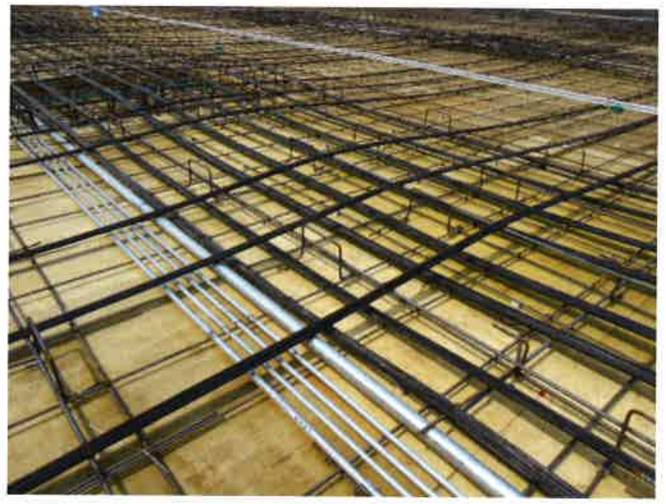


FIGURE 9.4-3 Unbonded Floor System (South Korea; Dailem P732)

FIGURE 9.4-4 Unbonded Floor System (Iceland; P733)

FIGURE 9.4-5 Stressing at Edge of Large Opening Made in an Unbonded Floor System (San Francisco; P734)

FIGURE 9.4-6 View of Tendons Re-anchored at the Edges of a Large Opening Made in an Unbonded Floor System (San Francisco; P735)

CHAPTER 10

STRESS LOSSES IN PRESTRESSING

Tendon Stressing at Slab Edge (P607)

10.1 OVERVIEW

The distribution of stress along a prestressed strand within concrete is non-uniform and decreases with time. The principal factors affecting the distribution of stress along a prestressing strand are:

- friction losses during stressing (jacking);
- retraction of strand as it is seated and locked against the anchorage device (seating loss);
- elastic shortening of concrete, elastic response of the prestressed member under applied loads;
- shrinkage of concrete;
- creep of concrete; and
- relaxation of prestressing steel.

Other factors, such as changes in temperature and flexing of a prestressed member under transient live loads also affect the stresses in a strand, but they do not necessarily lead to a permanent change in stress level and are not considered as stress losses.

The total prestress loss for unbonded, low-relaxation tendons is typically 20 percent of the jacking stress. In earlier designs a lump sum stress loss of 30 ksi (200 MPa) (14%) was assumed for several years for pre-tensioned members, since there is no friction loss in pretensioning. The development of low-relaxation strands and results of subsequent studies prompted a call for better estimates. A rigorous evaluation of stress losses is both time consuming and complex, however. Precise calculations for each tendon are not usually warranted in most residential and commercial buildings; studies have indicated that reliable solutions can be obtained with a number of simplifying assumptions. The following reviews the background to the sources of stress loss and offers a simple procedure for the estimate of stress losses for different conditions. The presentation of the procedure is followed by numerical examples. The procedure is based on the recommendations of ACI as referenced below.

ACI 318[1] refers to a study initiated by ACI/ASCE Committee 423, directed by Paul Zia and reported in Concrete International [Zia et al, 1979]. Research on friction losses and the background to the proposed procedures for their calculation are reported in numerous publications including several listed in the References at the end of this section.

It is assumed that the contributory factors such as friction, creep, and shrinkage are independent from one another. Hence, the loss due to each factor may be computed separately. The total stress loss in a tendon is the sum of the individually calculated losses.

10.2 DISTRIBUTION OF STRESS

The initial distribution of stress along a tendon is illustrated in Figs 10.2-1 for a beam with a continuous tendon stressed at both ends. It is assumed that the left end is stressed first. Part (b) shows the distribution of stress along the strand under jacking force and prior to locking off the strand. The jacking stress is commonly specified at $0.80f_{pu}$, where f_{pu} is the specified ultimate strength of strand. The smooth curve is a simplification of the actual distribution for illustration purposes however. The actual shape of the curve is determined by the tendon profile and friction parameters.

Part (c) of the figure shows the distribution after the strand is locked off at the left end of the beam. Observe that the initial stress is partially lost over a length of strand at the left end marked XL. This is due to the retraction of the strand at the stressing end while the wedges are being seated within the truncated cone of the anchorage device. Per ACI 318, the maximum permissible stress value immediately after lock-off and away from anchorage device is $0.74f_{pu}$. The maximum stress occurs at XL. The maximum permissible stress at the anchorage immediately after seating of the strand is $0.70f_{pu}$.

The seating loss, also referred to as anchorage set or draw-in, is typically 3/8 to 1/4 of an inch (6 to 9 mm). For short strands, and/or larger values of seating loss, the length XL may extend to the far end of the strand. Stressing rams with power seating capability will minimize the seating loss. Note that the retraction of the strand is resisted by the same friction forces that resisted the initial stressing. The stress diagram along length XL thus has the same gradient as the remainder of the curve, but in the opposite direction.

In most cases, jacking of the tendon at right end, part (d) of the figure, raises the stresses to about the mid-point of the tendon and the stress diagram will have a second peak at XR (part (e) of the figure). The distribution of stress immediately after the strand is seated at the right end is shown in part (e). Note that the lock-off stresses at the left and right ends are not generally the same unless the tendon is symmetrical about its mid-length.

The average initial stress is the average of this stress distribution. This value is used by some designers to calculate the stresses in unbonded post-tensioned structures at the transfer of post-tensioning. Transfer of post-tensioning refers to the loading condition immediately after stressing, prior to the application of live loading and the influences of long-term stress losses. It is also referred to as the "lock-off stress".

As long-term stress losses occur, the stress in strand is reduced along its length. Part (f) of the figure shows a schematic of the stress distribution after all losses have taken place. The following should be noted with respect to the final distribution of stress:

- ❖ Long-term stress losses along a tendon are not constant. Even under uniform geometry and exposure conditions, differences in concrete stress along a strand result in non-uniform losses. In the design of commercial buildings, however, it is common practice to calculate a representative long-term loss value for the

[1] ACI 318-11, Section R 18.6.1

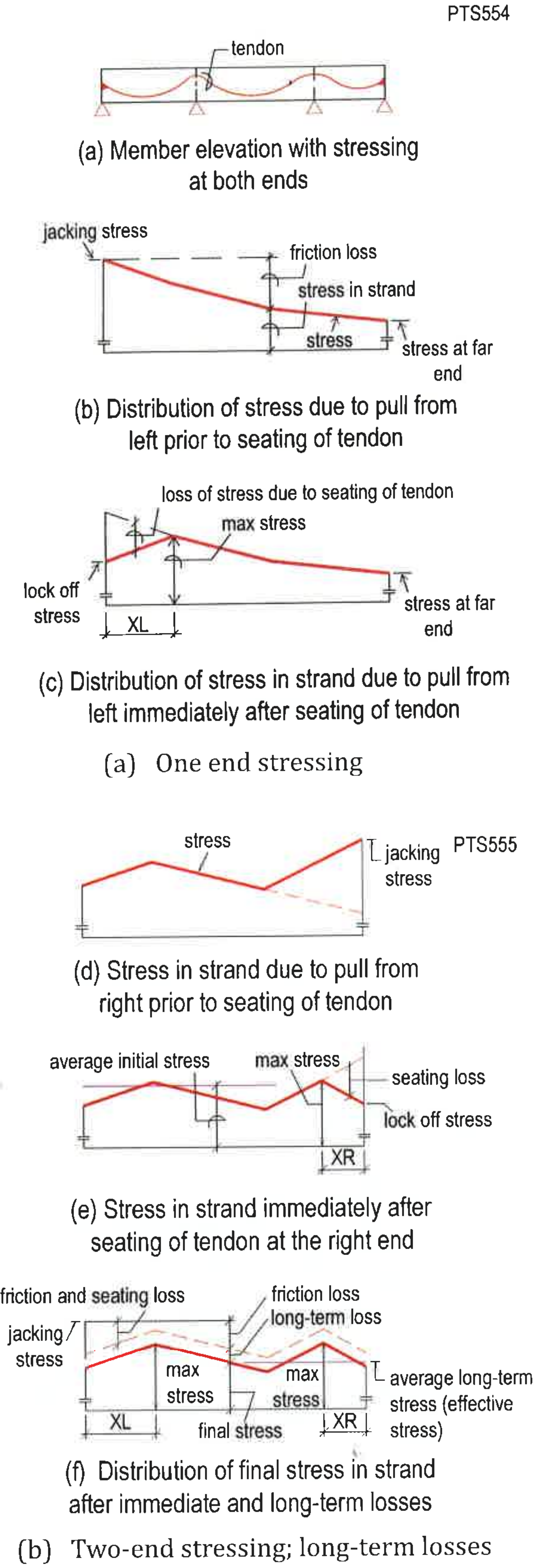


FIGURE 10.2-1 Friction and Long-Term Stress Losses

entire member when unbonded tendons are used. The average precompression in concrete is used to calculate the representative stress loss. The average precompression is calculated using the effective prestressing force and gross cross-section of concrete. In bonded (grouted) tendon construction, long-term losses are strictly a function of concrete strain at location of tendon along the length of member.

- ❖ Long-term stress losses are obviously a function of time. The relationships developed by the ACI/ASCE [Zia, P., et al, 1979] committee refer to a time at which over 90 percent of the losses have taken place. For common commercial buildings this period is between 2 and 2.5 years. The stress loss rates for shrinkage, creep and relaxation are not the same however. The curve shown in Fig. 10.2-2 may be used as a first approximation to estimate the combined stress losses for concrete at earlier ages. This diagram is compiled from the combined effects of shrinkage and creep using data from the reference [PCI, 1999].

The stress diagrams shown in Fig. 10.2-1 represent the maximum possible stress gradient attainable from the friction coefficients. The diagram is constructed with the maximum gradient at all points. With unbonded strands, flexing of the member due

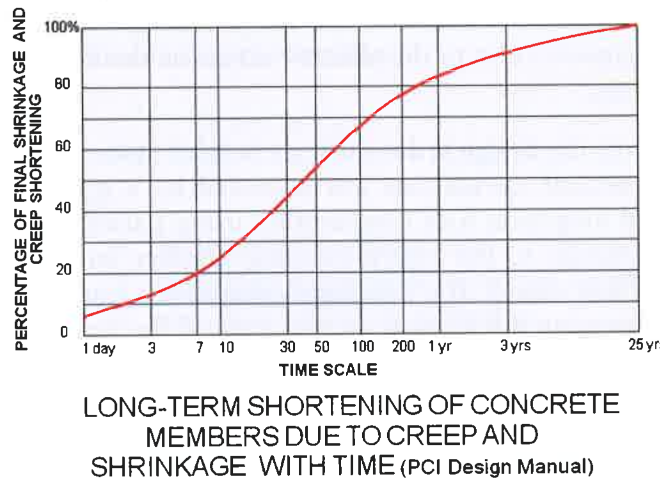


FIGURE 10.2-2 Long-Term Shortening of Concrete Members due to Creep and Shrinkage with Time[2] (P569)

[2] PCI Design Manual

to applied loading, temperature changes, shrinkage and creep can only reduce the stress gradient. Thus there could actually be a flattening of the diagram toward a more uniform stress distribution along the length of the tendon. This supports the premise for the use of "effective" force in design of post-tensioned members reinforced with unbonded tendons. There do not appear to be any conclusive studies that would quantify the extent of the stress redistribution however.

If a final "effective force" design approach is used, the outcome of the design is an effective force to be provided by post-tensioning. The effective force is the value that is shown on the structural drawings and in the calculations. The question of whether the effective force is based on average stresses, local stresses, or other considerations is not applicable during design.

At the shop (fabrication) drawing preparation phase, the "effective forces" must be replaced by the number of strands. In theory, the actual stresses in the strand at each location should be used to arrive at the number of strands required at that location. Because of the lack of information, and the complexity of this approach, however, in the North America practice an effective stress is typically used when designing commercial buildings with unbonded tendons. The effective stress is the average initial stress (Fig. 10-2-1(e)) minus a representative long-term stress loss value calculated for the entire member. Some engineers refer to the effective stress as the design stress.

When the design is done using a bonded system, the structural calculations are preceded by a friction and long-term loss computation using parameters particular to the post-tensioning supplier, such as friction values. The structural calculations can thus determine the number and location of the strands. In this case, the calculation of the design stress is of prime importance to the structural designer.

10.3 FRICTION AND SEATING LOSS CALCULATIONS

10.3.1 Stress Loss Due to Friction

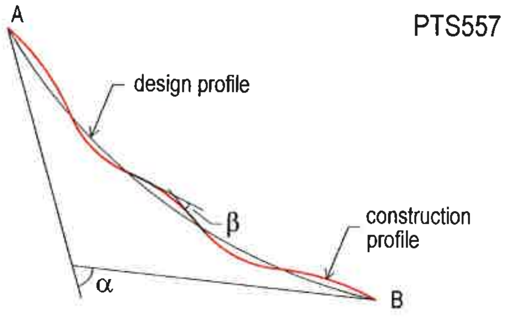
The stress at any point along a strand is related to the jacking stress through the following relationship:

$$P_x = P_j e^{-(\mu\alpha + Kx)} \qquad \text{(Exp 10.3-1)}$$

Where,

- P_x = stress at distance x from the jacking point;
- P_j = stress at jacking point;
- μ = coefficient of angular friction;
- α = total angle change of the strand in radians from the stressing point to distance x;
- x = distance from the stressing point; and
- K = wobble coefficient of friction expressed in radians per unit length of strand (rad/unit length[3]).

Figure 10.3.1-1 illustrates the design intended change in angle α and the unintended change in angle β dependent on construction practice. The unintended change in angle is the basis of the wobble coefficient (K).



α = intended angle change
β = unintended angle change

FIGURE 10.3.1-1 Angle Change in Tendon Profile

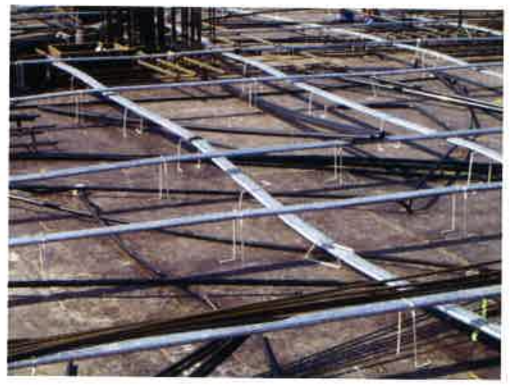


FIGURE 10.3.1-2 View of Tendon Layout Showing Wobble in Tendon (Tehran; P102)

[3] The dimension of the wobble coefficient K includes the coefficient of friction. K is μ (average of unintended change in angle per unit length of tendon)

Table 10.3.1-1, extracted from the ACI 318[4], gives friction coefficients for common strand and duct materials. The post-tensioning supplier should be consulted for friction coefficients of duct and coating materials not shown.

TABLE 10.3.1-1 Friction Coefficients for Post-Tensioned Tendons (T169)

			Wobble Coefficient K rad/ft ; (rad/m)	Curvature coefficient μ /radian
Grouted tendons in metal sheathing		Wire tendons	0.0010-0.0015 (0.0033-0.0049)	0.15-0.25
		High-strength bars	0.0001-0.0006 (0.0003-0.0020)	0.08-0.30
		7-wire strand	0.0005-0.0020 (0.0016-0.0066)	0.15-0.25
Unbonded tendons	Mastic coated	Wire tendons	0.0010-0.0020 (0.0033-0.0066)	0.05-0.15
		7-wire strand	0.0010-0.0020 (0.0033-0.0066)	0.05-0.15
	Pre-greased	Wire tendons	0.0003-0.0020 (0.0010-0.0066)	0.05-0.15
		7-wire strand	0.0003-0.0020 (0.0010-0.0066)	0.05-0.15

10. 3.2 Elongation

Figure 10.3.2-1(a) shows a typical post-tensioning tendon profile. When the jacking force, P_j, is applied at the stressing end, the tendon will elongate in accordance with the following formula:

$$\Delta = \int \frac{P_x}{AE_s} dx \qquad \text{(Exp 10.3.2-1)}$$

Where,

- A = cross-sectional area of the tendon;
- dx = the element of length along tendon;
- E_s = modulus of elasticity of the prestressing steel (typically taken as either 28000 or 28500 ksi) (193054 MPa or 196502 MPa) ;
- P_x = tendon force at distance x from the jacking end; and
- Δ = calculated elongation.

This elongation will be resisted by friction between the strand and its sheathing or duct, however. As a result of this friction, there will be a drop in the force in the tendon with distance from the jacking end. The friction is comprised of two effects: curvature friction which is a function of the tendon's profile, and wobble friction which is the result of minor horizontal or vertical angular deviations from the design-intended profile. Curvature friction is greatest where there are short spans with fairly large changes in profile.

10.3.3 Seating Losses due to Seating of Strand

After they are stressed, tendons are typically anchored with two-piece conical wedges . The strand retracts when it is released and pulls the wedges into the anchorage device; this forces the wedges together and locks the strand in place. The stress loss due to seating is somewhat hard to calculate because the loss in elongation is fairly small (it depends on both the jack and jacking procedure.) In addition, the loss in elongation (referred to as anchor set, or draw-in) is resisted by friction much as the elongation itself is resisted by friction.

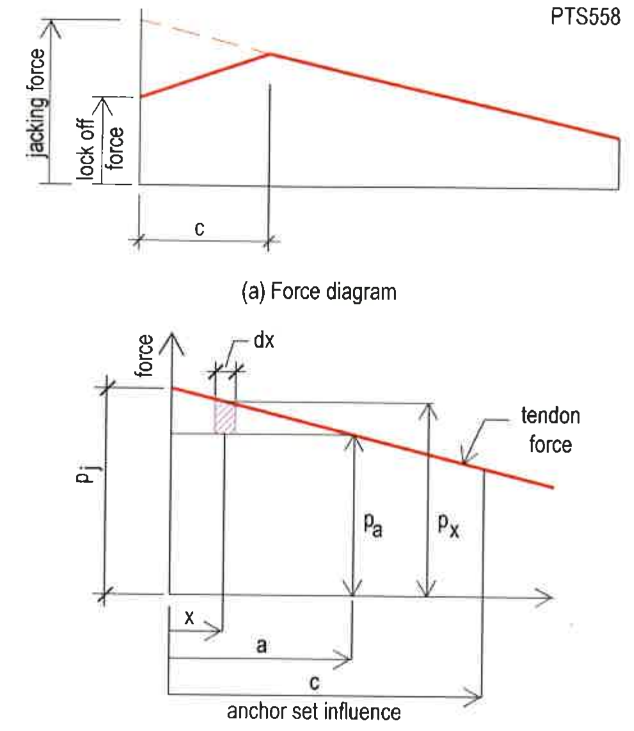
Calculation of the stress loss is typically done as an iterative process. Refer to Fig. 10.3.3-2a. After anchor set, the region of stress distribution affected by the loss in stress is the mirror image of the associated stress curve under jacking force. The extent of the influence of anchor set is shown as "c." For the calculation of distance "c" an anchor set influence length "a" is chosen (Fig. 10. 3.3-2b). An elongation Δ_a for the selected distance "a" is calculated using the formula:

$$\Delta_a = \frac{1}{AE_s} \int (P_x - P_a)\, dx$$

Where,

- A = tendon cross sectional area;
- dx = element of distance along tendon length;
- E_s = modulus of elasticity of tendon;
- P_a = force in tendon under jacking stress at the assumed anchor set distance "a";
- P_x = force in tendon at distance "x" from the stressing end; and
- Δ_a = elongation associated with the assumed anchor set influence length "a".

[4] ACI 318-08, Table R18.6.2



(a) Force diagram

(b) Anchor set calculation schematic

FIGURE 10.3.3-2 Anchor-Set Distance Diagram

The anchor set length is adjusted until the calculated Δ_a is reasonably close to the seating loss. The iterative approach is oftentimes adopted, since in the general case the friction loss curve does not follow a straight line. The integral is carried out for each stressing end.

The average stress is calculated as the area under the stress diagram divided by the length of the tendon. Note that the slope of the "post-seating" stress line is the inverse of the initial stress loss line. The elongation for the first stressing is the average stress in the tendon after the first stressing, divided by the modulus of elasticity of the strand. The elongation for the second stressing is the average stress in the tendon divided by the modulus of elasticity, minus the first elongation.

10.4 LONG-TERM STRESS LOSS ESTIMATE

For common structures and conditions, simplified expressions are used to estimate the stress losses due to prestressing. The expressions are based on the work of ACI-ASCE Committee 423 [1979], PCI [1999]. The expressions enable the designer to estimate the various types of prestress loss, rather than using a lump sum value. It is believed that these equations, intended for practical design applications, provide fairly realistic values of normal design conditions. For unusual design situations and special structures, more detailed analysis may we warranted [Aalami, B. O., 1998].

$$TL = ES + CR + SH + RE \qquad \text{(Exp 10.4-1)}$$

Where,

CR = stress loss due to creep:
ES = stress change due to elastic deformation;
RE = stress loss due to relaxation in prestressing steel;
SH = stress loss due to shrinkage of concrete; and
TL = total loss of stress.

10.4.1 Elastic Deformation of Concrete (ES)

This is often referred to as "Elastic Shortening." Elastic shortening refers to the shortening of the concrete member as the post-tensioning force is applied. If there is only one tendon in a member, there will be no loss due to elastic shortening since the elastic shortening will have occurred before the tendon is locked into place. Generally, however there will be several tendons in a member. As each tendon is tensioned, there will be a loss of prestress in the previously tensioned tendons due to the elastic shortening of the member.

Since an unbonded tendon can slide within its sheathing, it typically does not experience the same stress-induced strain changes as the concrete surrounding it. For this reason, the average compressive stress in the concrete, f_{cpa}, is typically used to calculate prestress losses due to elastic shortening and creep for unbonded tendons. This relates these prestress losses to the average member strain rather than the strain at the point of maximum moment. For grouted tendons, the loss from shortening of a member before grouting is the same as for unbonded tendons. However, subsequent to grouting, there will be an additional local change in stress due to bond of tendon to adjoining concrete. Since selfweight can be considered a permanent load, the change of stress due to elastic response of concrete under selfweight becomes part of "*ES*."

The equation given for calculating elastic shortening for unbonded tendons is:

$$ES = K_{es}\left(\frac{E_s}{E_{ci}}\right) f_{cpa} \qquad \text{(Exp 10.4.1-1)}$$

Where,

E_S = is the elastic modulus of the prestressing steel;
E_{ci} = is the elastic modulus of the concrete at time of prestress transfer;
K_{es} = 1.0 for pre-tensioned members;
K_{es} = 0.5 for post-tensioned members when tendons are tensioned in sequential order to the same tension. With other post-tensioning procedures, K_{es} may vary from 0 to 0.5; and
f_{cpa} = average compressive stress in the concrete along the length of the member at the centroid of the section.

At the time they are stressed, the ducts in which post-tensioned bonded tendons are housed have not been grouted. Thus, the elastic shortening equations for unbonded tendons would apply to these tendons as well. Subsequent to grouting, there will be additional elastic change in tendon length due to flexing of member.

For pre-tensioned bonded members the following relationship applied.

$$ES = K_{es}\left(\frac{E_s}{E_{ci}}\right) f_{cir} \qquad \text{(Exp 10.4.1-2)}$$

Where,

K_{es} = 1.0 for pre-tensioned members;
f_{cir} = net compressive stress in concrete at center of gravity of tendons immediately after prestress has been applied to concrete;

$f_{cir} = K_{cir} f_{cpi} - f_g$
K_{cir} = 0.9 for pre-tensioned members;
K_{cir} = 1.0 for post-tensioned members;
f_{cpi} = stress in concrete at center of gravity of tendons due to prestressing forces immediately after prestress has been applied;

$$f_{cpi} = \left(\frac{P_i}{A_g} + \frac{P_i e^2}{I_g}\right) \qquad \text{(Exp 10.4.1-3)}$$

Where,

P_i = initial prestressing force (after anchorage seating loss);
A_g = area of gross concrete section at the cross section considered;
e = eccentricity of center of gravity of tendons with respect to center of gravity of concrete at the cross section considered;
I_g = moment of inertia of gross concrete section at the cross section considered; and
f_g = stress in concrete at center of gravity of tendons due to weight of structure at time prestress is applied (positive if tension).

$$f_g = \frac{M_g e}{I_g} \qquad \text{(Exp 10.4.1-4)}$$

Where,

M_g = bending moment due to dead weight of prestressed member and any other permanent loads in place at time the prestressed member is lifted off its bed. There can be an increase in tendon force at the section considered, if pre-tensioning is not adequate to give the member an upward camber (tension due to selfweight (f_g) greater than compression due to prestressing (f_{cpi})).

Hence

$$f_{cir} = K_{cir}\left(\frac{P_i}{A_g} + \frac{P_i e^2}{I_g}\right) - \frac{M_g e}{I_g} \qquad \text{(Exp 10.4.1-5)}$$

10.4.2 Creep of Concrete (*CR*)

Over time, the compressive stress induced by post-tensioning causes a shortening of the concrete member. This phenomenon, the increase in strain due to a sustained stress, is referred to as creep. Loss of prestress due to creep is proportional to the net permanent compressive stress in the concrete. The initial compressive stress induced in the concrete at transfer is subsequently reduced by the tensile stress resulting from self-weight and superimposed dead load moments.

For members with **unbonded** tendons, the equation is:

$$CR = K_{cr}\left(\frac{E_s}{E_c}\right)f_{cpa} \qquad \text{(Exp 10.4.2-1)}$$

For members with **bonded** and **pre-tensioned** tendons, the equation is:

$$CR = K_{cr}\left(\frac{E_s}{E_c}\right)(f_{cir} - f_{cds}) \qquad \text{(Exp 10.4.2-2)}$$

Where,

E_c = elastic modulus of the concrete at 28 days;

f_{cds} = stress in the concrete at the center of tension of the tendons due to all sustained loads that are applied to the member after it has been stressed; and

K_{cr} = maximum creep coefficient; 2.0 for normal weight concrete; 1.6 for sand-lightweight concrete.

The difference in the equations is due to the fact that unbonded tendons do not experience the same strains as the surrounding concrete. The prestress loss due to creep is thus more logically related to the average stress in the concrete section. With bonded tendons however, once the duct is grouted the shortening of the concrete member due to creep will result in a comparable shortening (loss of elongation) in the tendon. The same applies to pre-tensioned members.

10.4.3 Shrinkage of Concrete (*SH*)

In the calculation of prestress losses, shrinkage is considered to be entirely a function of water loss. Shrinkage strain is thus influenced by the member's volume/surface ratio and the ambient relative humidity. The effective shrinkage strain, ε_{sh} is obtained by multiplying the basic ultimate shrinkage strain, $(\varepsilon_{sh})_{ultimate}$, taken as 550 x 10^{-6} for normal conditions, by the factors (1-0.06 V/S) to allow for member dimension, and (1.5 - 0.015RH) for ambient humidity. Where detailed information is available, the basic ultimate shrinkage value 550 micro strain, can be adjusted to match the properties of concrete used.

$$\varepsilon_{sh} = 550\times10^{-6}\left(1-0.06\frac{V}{S}\right)(1.5-0.015RH)$$

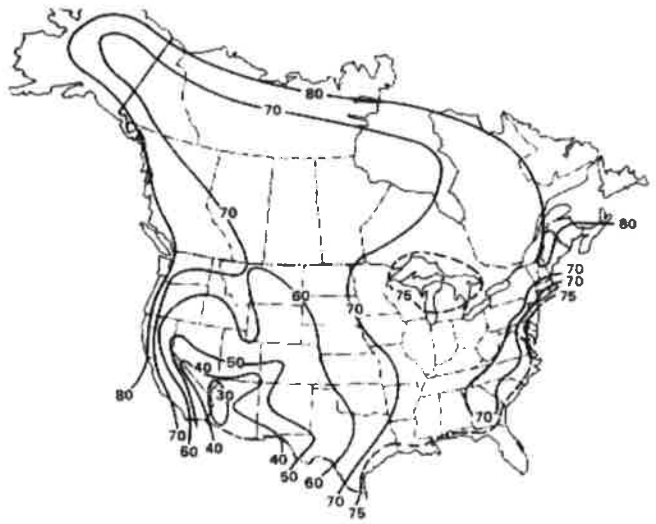


FIGURE 10.4.3-1 Annual Average Ambient Relative Humidity, Percent[5] (P610)

TABLE 10.4.3-1 Shrinkage Coefficient K_{sh} (T170)

Days *	1	3	5	7	10	20	30	60
K_{sh}	0.92	0.85	0.80	0.77	0.73	0.64	0.58	0.45

*Days refer to the time from the end of moist curing to the application of prestressing.
For stressing more than 60 days after curing, assume 0.45

$$\varepsilon_{sh} = 8.2\times10^{-6}\left(1-0.06\frac{V}{S}\right)(100-RH)$$

The equation for losses due to shrinkage is:

$$SH = 8.2\times10^{-6}K_{sh}E_s\left(1-0.06\frac{V}{S}\right)(100-RH)$$

(US Units) (Exp 10.4.3-1 US)

$$SH = 8.2\times10^{-6}K_{sh}E_s\left(1-0.00236\frac{V}{S}\right)(100-RH)$$

(SI Units) (Exp 10.4.3-1 SI)

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Where,

V/S = volume to surface ratio;

RH = relative humidity (percent), see Fig. 10.4.3-1 for USA; and

K_{sh} = a factor that accounts for the amount of shrinkage that will have taken place before the prestressing is applied.

For post-tensioned members, K_{sh} is taken from Table 10.4.3-1.

For pre-tensioned members, K_{sh} = 1;

In structures that are not moist cured, K_{sh} is typically based on the time when the concrete was cast. It should be noted that in most structures, the prestressing is applied within five days of casting the concrete, whether or not it is moist-cured.

If the ultimate shrinkage value of the concrete is different from 550 micro-strain used in the above relationships, the calculated stress loss must be adjusted by the following coefficient.

$$SH_{adjusted} = SH\left[\frac{(\varepsilon_{sh})_{ultimate}}{550}\right] \qquad \text{(Exp 10.4.3-2)}$$

Where $(\varepsilon_{sh})_{ultimate}$ is the ultimate shrinkage strain of the concrete used.

10.4.4 Relaxation of Prestressing Steel (RE)

Relaxation is defined as a gradual decrease of stress in a material under constant strain. In the case of steel, it is the result of a permanent alteration of the grain structure. The rate of relaxation at any point in time depends on the stress level in the tendon at that time. Because of other prestress losses, there is a continual reduction of the tendon stress which causes a corresponding reduction in the relaxation rate.

The equation given for prestress loss due to relaxation of the tendons is:

$$RE = \left[K_{re} - J\left(SH + CR + ES\right)\right]C \qquad \text{(Exp 10.4.4-1)}$$

Where, K_{re} and J are a function of the type of steel and C is a function of both the type of steel and the initial stress level in the tendon (f_{pi}/f_{pu}).

Table 10.4.4-1 gives values of K_{re} and J for different types of steel. The factor J accounts for the reduction in tendon stress due to other losses. As can be seen, the relaxation of low-relaxation strands is approximately one-quarter that of stress- relieved strands.

TABLE 10.4.4-1 Stress Relaxation Constants K_{re} and J (T171)

	Grade and type*	K_{re} US (psi)	K_{re} SI (MPa)	K_{re} MKS (kg/cm²)	J
Stress relieved	270 strand or wire	20000	137.90	1406.14	0.15
	250 strand or wire	18500	127.55	1300.68	0.14
	240 wire	17600	121.35	1237.40	0.13
	235 wire	17600	121.35	1237.40	0.13
	160 bar	6000	41.37	421.84	0.05
	145 bar	6000	41.37	421.84	0.05
Low relaxation	270 strand	5000	34.47	351.54	0.040
	250 wire	4630	31.92	325.52	0.037
	240 wire	4400	30.34	309.35	0.035
	235 wire	4400	30.34	309.35	0.035

*In accordance with ASTM A416-74, ASTM A421-76, ASTM A722-75

Table 10.4.4-2 gives values for *C*. The values for stress-relieved and low-relaxation tendons are different because the yield stress for low relaxation tendons is higher than that of the same grade stress-relieved tendons. Although ACI allows a stress of $0.74 f_{pu}$ along the length of the tendon immediately after prestress transfer, the stress at post-tensioning anchorages and couplers is limited to $0.70 f_{pu}$. In the absence of more exact calculations, the ratio (f_{pi}/f_{pu}) is typically taken as 0.70 for unbonded post-tensioning. With very short tendons however, the loss due to anchor set may be such that (f_{pi}/f_{pu}) is considerably lower.

TABLE 10.4.4-2 Stress Relaxation Constant C (T172)

f_{pi}/f_{pu}	Stress Relieved strand or wire	Stress relieved bar or Low Relaxation strand or wire
0.80		1.28
0.79		1.22
0.78		1.16
0.77		1.11
0.76		1.05
0.75	1.45	1.00
0.74	1.36	0.95
0.73	1.27	0.90
0.72	1.18	0.85
0.71	1.09	0.80
0.70	1.00	0.75
0.69	0.94	0.70
0.68	0.89	0.66
0.67	0.83	0.61
0.66	0.78	0.57
0.65	0.73	0.53
0.64	0.68	0.49
0.63	0.63	0.45
0.62	0.58	0.41
0.61	0.53	0.37
0.60	0.49	0.33

For (f_{pi}/f_{pu}) outside the values given in this table, the following is assumed:

Stress-relieved strand and wire:

For $0.00 < (f_{pi}/f_{pu}) < 0.60$, C = linear between 0 and 0.49

For $0.75 < (f_{pi}/f_{pu}) < 0.95$, $C = 1.45$

Stress-relieved bar and low-relaxation strand and wire:

For $0.00 < (f_{pi}/f_{pu}) < 0.60$, C = linear between 0 and 0.33

For $0.80 < (f_{pi}/f_{pu}) < 0.95$, $C = 1.36$

10.5 EXAMPLES

10.5.1 Friction and Long-Term Stress Losses of an Unbonded Post-Tensioned Slab

Structure: Slab of a parking structure resting on parallel beams. The slab is post-tensioned with unbonded tendons providing an average precompression of 250 psi (1.72 MPa) after all losses.

❖ Geometry

Six span one way slab spanning 18'-0" (5.49 m) between 14 in. x 34 in. (356x864 mm) cast in place concrete beams (Fig 10.5.1-1).

Thickness of slab = 5 in (127 mm)

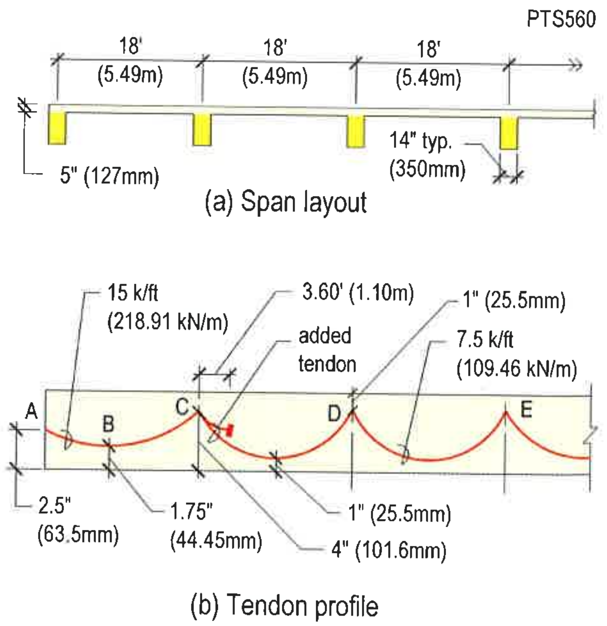


FIGURE 10.5.1-1 Slab Dimensions and Prestressing

TABLE 10.5.1-1 Summary of Angle Change and Friction Losses (T173)

Location	δx,ft (m)	X,ft (m)	δα (rad)	α (rad)	μα+Kx	$e^{-\mu\alpha+Kx}$	P_x,ksi (MPa)	Loss, ksi (MPa)
A	0	0	0	0	0	1	216 (1489.28)	0
B	6.59 (2.01)	6.59 (2.01)	0.019	0.019	0.011	0.989	213.62 (1472.87)	2.38 (16.41)
C	11.41 (3.48)	18.00 (5.49)	0.033	0.052	0.029	0.971	209.74 (1446.12)	3.88 (26.75)
D	18.00 (5.49)	36.00 (10.97)	0.056	0.108	0.058	0.944	203.90 (1405.85)	5.84 (40.27)
E	18.00 (5.49)	54.00 (16.46)	0.056	0.164	0.087	0.917	198.07 (1365.65)	5.83 (40.20)

❖ Material Properties

Concrete:

Compressive strength (28 days), f'_c = 4000 psi (27.58 MPa)

Weight = 150 pcf (2403 kg/m³)

Modulus of Elasticity = 3604 ksi (24849 MPa)

Age of concrete at stressing = 3 days

Compressive strength at stressing, f'_{ci} = 1832 psi (12.63 MPa)

Prestressing:

Low relaxation, unbonded System

Strand diameter =½ in (13 mm)

Strand area =0.153 in² (98 mm²)

Modulus of Elasticity = 28000 ksi (193054 MPa)

Coefficient of angular friction, μ = 0.07

Coefficient of wobble friction, *K* =0.0014 rad/ft (0.0046 rad/m)

Ultimate strength of strand, f_{pu} = 270 ksi (1862 MPa)

Ratio of jacking stress to strand's ultimate strength = 0.8

Anchor set = 0.25 in (6.35 mm)

❖ Loading:

Dead load = self weight + 5 psf (allowance for curbs, lighting, drainage etc.)

$$= \left[(5/12)\times 150\right] + 5 = 68 \text{ psf } (3.26 \text{ kN/m}^2)$$

Live load = 50 psf (2.39 kN/m²)

A. Friction and Seating Loss

i. Friction Loss

Considering only the left half span since the tendon is symmetrical about the mid length and stressed from both ends.

Stress at distance x from the jacking point,

$$P_x = P_j e^{-(\mu\alpha + Kx)}$$

where,

P_j = stress at jacking point;
= $0.8 \times f_{pu} = 0.8 \times 270 = 216$ ksi (1489.28 MPa);

x = distance from the stressing point; and

α = change of angle in strand (radians) from the stressing point to distance x.

Calculation of α:

Span 1:

$$C = L\frac{\sqrt{a/b}}{1+\sqrt{a/b}}$$

$$C = 18\frac{\sqrt{0.75/2.25}}{1+\sqrt{0.75/2.25}} = 6.59 \text{ ft}(2.01 \text{ m})$$

AB:

$$\alpha = 2e/L = 2(0.75/12)/6.59 = 0.019 \ rad$$

Where "*e*" is the drape and " *L* " is the distance along the member.

BC:

$$\alpha = 2e/L = 2(2.25/12)/11.41 = 0.033 \ rad$$

Span 2:
CD:
$\alpha = 4e / L = 4(3/12)/18 = 0.056\ rad$

Span 3:
DE:
$= 4e / L = 4(3/12)/18 = 0.056\ rad$

The following table summarizes the stress calculations at supports and first span midpoint, followed by detailed calculation for midspan of first span.

Detailed Calculation at point B:

x = 6.59 ft (2.01 m)
α = 0.019 rad,
μ = 0.07
K = 0.0014 rad/ft (0.0046 rad/m)
P_S = 0.8 × f_{pu} = 0.8 × 270
= 216 ksi (1489.28 MPa)

$P_{mid} = P_s\ e^{-(\mu\alpha + Kx)} = 216\ e^{-(0.07\times0.019+0.0014\times6.59)}$
= 213.62 ksi (1472.87 MPa)

Loss = 216 -213.62 = 2.38 ksi (16.41 MPa)

ii. Seating Loss

Stress loss due to seating can be calculated from the following relationship,

$$a = \frac{1}{E_s}\int(\mathit{final\ stress} - \mathit{initial\ stress})\,dx$$

Where,

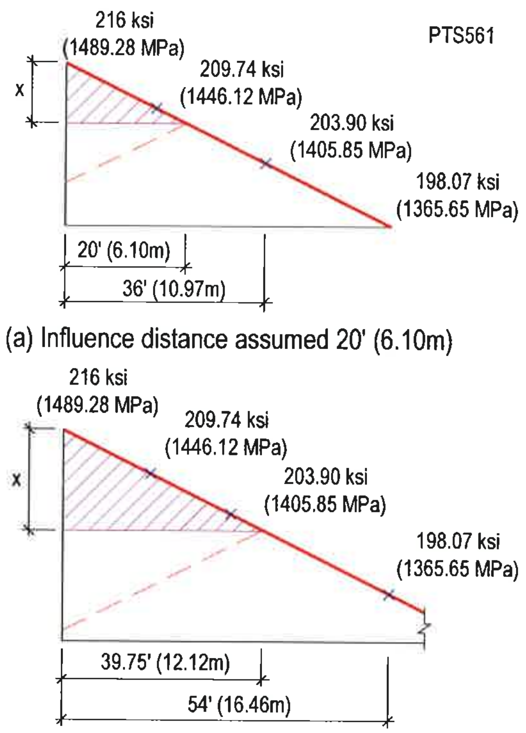
a = anchor set = 0.25 in (6.35 mm)
E_s = modulus of elasticity of tendon
= 28000 ksi (193054 MPa)

Consider the section "x" at 20 ft (6.100 m) from the stressing point and calculate the seating loss. (Fig 10.5.1-2a)
Assume that the stress loss is linear along the span.

$(216 - 203.90)/36 = x/20$
x = 6.72 ksi (46.33 MPa)

Seating loss, $a = (2\times6.72\times20\times12)/(2\times28000) = 0.06\ \text{in}(1.52\ \text{mm})$

This is much less than the given seating loss, 0.25 in



(a) Influence distance assumed 20' (6.10m)

(b) Influence distance assumed 39.75' (12.12m)

FIGURE 10.5.1-2 Seating Loss Influence Distance

Consider the section "x" at 39.75 ft (12.116 m) from the stressing point and calculate the seating loss. (Fig 10. 5.1-2b)

(216 – 198.07) / 54 = x/39.75
x = 13.20 ksi

Seating loss,

$a = (2\times13.20\times39.75\times12)/(2\times28000) =$
$= 0.23$ in (5.84 mm) $\approx$ 0.25 in (6.35 mm)

Maximum stress in the tendon =216 – 13.20
= 202.80 ksi (1398.27 MPa)

B. Long-Term Loss

Long-Term loss calculation for first span:

❖ Elastic Shortening:

$$ES = K_{es}\left(\frac{E_s}{E_{ci}}\right)f_{cpa}$$

f_{cpa} = 250 psi (1.72 MPa)
E_{ci} = 2440 ksi (16823 MPa)
E_s = 28000 ksi (193054 MPa)
ES =
$0.5\times(28000/2440)\times0.250 = 1.434$ ksi (9.89 MPa)

TABLE 10.5.1-2 Summary of Long-Term Stress Losses (T174)

	ksi	MPa	%
Elastic shortening (ES)	1.43	9.89	11
Creep (CR)	3.32	22.88	24
Shrinkage (SH)	3.11	21.43	23
Relaxation (RE)	5.72	39.41	42
Total	13.58	93.60	100

❖ Shrinkage of Concrete:

$$SH = 8.2\times10^{-6}K_{sh}E_s\left(1-0.06\frac{V}{S}\right)(100-RH)$$

V/S = 2.50 in (63.5 mm)
RH = 80 %
K_{sh} = 0.85 (from Table 10.4.3-1 for 3 days)
SH =
$8.2\times10^{-6}\times0.85\times28000(1-0.06\times2.5)(100-80)$
= 3.318 ksi (22.88 MPa)

❖ Creep of Concrete:

$$CR = K_{cr}\left(\frac{E_s}{E_c}\right)f_{cpa}$$

E_c = 3604 ksi (24849 MPa)
CR = $1.6\times(28000/3604)\times0.250$ = 3.108 ksi (21.43 MPa) for sand lightweight concrete

❖ Relaxation of Strands:

$$RE = [K_{re} - J(SH + CR + ES)]C$$

For 270 ksi low relaxation strand, from Table 10.4.4-1:

K_{re} = 5 ksi (34.47 MPa)
J = 0.04
f_{pi} = 213.62 ksi (1473 MPa)
(considering midspan of first span as a representative point)

f_{pu} = 270 ksi (1861.60 MPa)
f_{pi}/f_{pu} = 213.62/270 = 0.79

From Table 10.4.4-2: C= 1.22

RE $= [5-0.04(1.434 + 3.318 + 3.108)]1.22 =$
= 5.716 ksi (39.41 MPa)

Total Stress Losses =1.434 +3.318 +3.108 +5.716 =
= 13.576 ksi (93.60 MPa)

10.5.2 Friction and Long-Term Stress Losses of a Beam Reinforced with Bonded Post-Tensioning

The friction and elongation calculation procedures are identical to the case of unbonded tendons treated in the preceding example. Hence, the friction and seating loss calculations are not repeated here. The long-term loss calculation, however, differs significantly, since the stress loss at a point in bonded tendons is tied to the strain in concrete adjacent to the tendon at the same point.

Structure

❖ Geometry

Two span beam with grouted post-tensioning as shown in Fig. 10.5.2-1 in elevation and in Fig. 10.5.2-2 in section.

❖ Material Properties

Concrete:
Compressive 28 day strength, f'_c = 27.58 MPa (4000 psi)
Weight = 2403 kg/m³ (150 pcf)
Modulus of Elasticity = 24849 MPa (3604 ksi)
Age of concrete at stressing = 3 days
Compressive strength at stressing, f'_{ci} = 20 MPa (2901 psi)

Prestressing:
Post-tensioning is provided by a single tendon consisting of 8-13 mm diameter low-relaxation strands stressed at one end

Strand diameter = 13 mm (½ in)
Strand area = 98 mm² (0.153 in²)
Modulus of Elasticity = 193000 MPa (27992 ksi)
Coefficient of angular friction, μ = 0.20
Coefficient of wobble friction, K = 0.0002 rad/m (0.000061 rad/ft)
Ultimate strength of strand, f_{pu} = 1862 MPa (270 ksi)
Ratio of jacking stress to strand's ultimate strength = 0.8
Anchor set = 6 mm (0.24 in)

❖ Loading

Selfweight of beam for 6 m tributary = 26 kN/m (1.78 k/ft)

Superimposed sustained load in addition to selfweight after the beam is placed in service (1.02 kN/m^2) = 2.63 kN/m (0.18 k/ft)

Live load is generally not considered for the long-term stress loss in grouted tendons.

It is required to calculate the long-term stress losses in the prestressing tendons at midpoint of the first span and over the interior support.

The long-term stress loss calculation consists of the following steps:

1. Determine the initial stress in tendon at midspan and over second support. Use a software or hand calculations.
2. Determine the bending moments and stresses at midspan and over the first support due to selfweight and the superimposed sustained loading. Also, calculate stresses due to post-tensioning tendons.
3. Use the relationships given in this Chapter to calculate long-term stress losses.

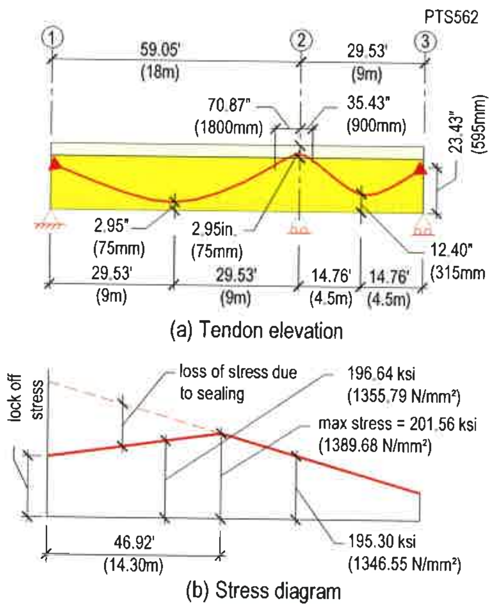


FIGURE 10.5.2-1 Tendon Profile and Force

PTS611

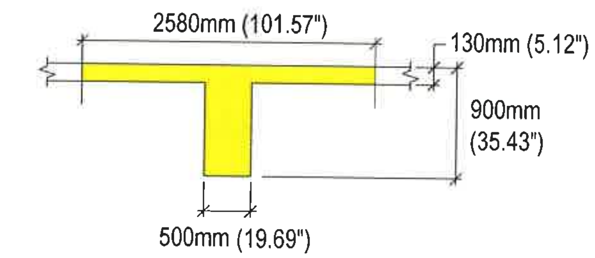
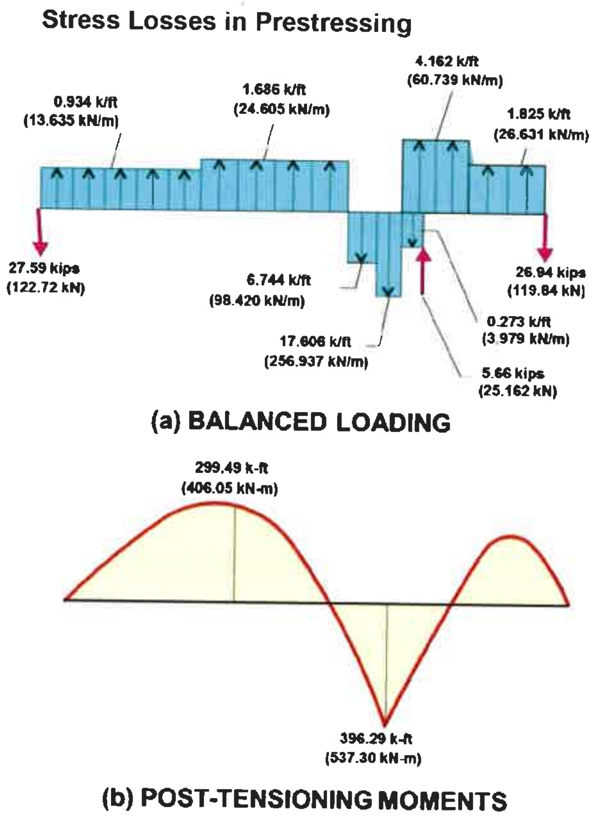


FIGURE 10.5.2-2 Section



POST-TENSIONING LOADS AND MOMENTS

FIGURE 10.5.2-3 Post-Tensioning Loads and Moments (P611)

A- Calculation of initial stress in tendon: The initial tendon stresses (f_{pi}) after anchor set may be read off from Fig 10. 5.2-1:

At midspan 1355.79 MPa (196.64 ksi)
At support 1346.55 MPa (195.30 ksi)

B- Bending moments and stresses at required points: The section properties of the beam are:

Cross sectional area A= 720400 mm^2 (1116.62 in^2)
Second moment of area I = 5.579*10^{10} mm^4 (134036 in^4)
Neutral axis to bottom fiber Y_b = 595.0 mm (23.43 in)
Neutral axis to top fiber Y_t = 305.0 mm (12.01 in)

Neutral axis to height of strand
At midspan c = 595-75 = 520.0 mm (20.47 in)
At support c = 305-75 = 230.0 mm(9.06 in)

The distribution of bending moments due to selfweight of the beam is obtained using a computer program[6]. Balanced loads from post-tensioning and post-tensioning moments are shown in Fig. 10.5.2-3.

Moment at midspan 658.34 kN-m (485.57 k-ft)
Moment at support -789.29 kN-m (-582.15 k-ft)

Stresses in concrete at center of gravity of tendons (f_g) due to weight of structure at time of stressing are calculated at height of tendon CGS[7]. This is a hypothetical point for concrete, as in the general case there is no concrete at CGS of tendons. The tendon spans between the supports, and is profiled such as to provide uplift against the weight of the beam on formwork. Selfweight of concrete is considered as a permanent load and the strain due to elastic deformation resulting from selfweight is considered as part of long-term stress change. Stresses from post-tensioning and selfweight contribute to the long-term changes in tendon stress.

Stress f_g at midspan:

$= \left(658.34\times10^6\times520\right)/\left(5.579\times10^{10}\right) =$
= 6.14 MPa (0.89 ksi)(tension)

Stress f_g at support:

$= \left(789.29\times10^6\times230\right)/\left(5.579\times10^{10}\right) =$
= 3.25 MPa (0.47 ksi) (tension)

Stresses due to superimposed sustained loading f_{cds} may be prorated from the selfweight stresses:

Stress f_{cds} at midspan =
= (2.63/26) × 6.14 = 0.62 MPa (0.0.09 ksi) (tension)

Stress f_{cds} at support =
=(2.63/26) × 3.25 = 0.33 MPa (0.0.05 ksi) (tension)

The initial stress in concrete is calculated from the balanced loading immediately after the tendon is seated. In other words, before long-term losses take place. The values of the balanced loading and the associated post- tensioning moments are given in Fig. 10.5.2-2.

At midspan M_b = -406.05 kN-m (-299.49 k-ft)
At support M_b = 537.30 kN-m (396.29 k-ft)

Initial concrete stress due to post-tensioning f_{cpi}:

At midspan:
The initial post-tensioning force is

P_{pi} = 8×98×1355.79 / 1000 = 1062.94 kN (238.96 kips)

$$f_{cpi} = \left(\frac{P_{pi}}{A} + \frac{M_b}{I}c\right) =$$

$$= \left(\frac{1062.94\times10^3}{720400} + \frac{406.05\times10^6\times520}{5.579\times10^{10}}\right) =$$

$$= 5.26\text{ MPa}\left(763\text{ psi}\right)\left(C\right)$$

At support:
The initial post-tensioning force

P_{pi} = 8×98×1346.55 / 1000 = 1055.70 kN (237.33 kips)

$$f_{cpi} = \left(\frac{1055.70\times10^3}{720400} + \frac{537.30\times10^6\times230}{5.579\times10^{10}}\right) =$$

$$= 3.68\text{ MPa}\left(534\text{ psi}\right)\left(C\right)$$

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6 ADAPT PT program for post-tensioned floor systems and beam frames www.adaptsoft.com
7 Center of Gravity of Steel

C. Calculation of long-term stress losses

i. At Mid-span

❖ Elastic Shortening:

$$ES = K_{es}\left(\frac{E_s}{E_{ci}}\right)f_{cir}$$

$K_{es} = 0$ (all strands are pulled and anchored simultaneously) ; hence,
$ES = 0$ MPa (0 psi)

Observe that, in this example, the long-term losses due to elastic shortening are zero since all the strands are stressed and anchored simultaneously.

❖ Creep of Concrete:
For the calculation of losses due to creep, the initial stress in concrete f_{cir} will be calculated with both the selfweight and the sustained superimposed loadings considered as active. Hence,

f_{cir} = 0.88 MPa (128 psi) (Tension) from elastic shortening calculations

f_{cir} = 5.26 -6.14 =-0.88 MPa(-128 psi) (Tension)

f_{cds}= 0.62 MPa (90 psi) (Tension) from stress calculations

$$CR = K_{cr}\left(\frac{E_s}{E_c}\right)(f_{cir} - f_{cds})$$

K_{cr} = 1.6
E_c = $4700\times 27.58^{1/2}$ = 24683 MPa (3580 ksi)
$(f_{cir} - f_{cds})$= -0.88 - 0.62 = -1.50 MPa (-218 psi) (Tension)

Observe that the net stresses $(f_{cir}$ - $f_{cds})$ are tensile. Stress loss due to creep is associated with compressive stresses only. A negative sum is substituted by zero. In fact, it results in an increase in stress. Therefore,

$CR = 1.6\times(193000/24683)\times 0 \;\; = 0$ MPa

❖ Shrinkage of Concrete
The relationship used for shrinkage is:

$$SH = 8.2\times10^{-6}K_{sh}E_s\left(1-0.00236\frac{V}{S}\right)(100-RH)$$

Where,
K_{sh} = 0.85 (for stressing at 3 days - Table 10.4.3-1); and
RH = 0.70 (given relative humidity).
V/S = volume to surface ratio
$= (2580\times130+500\times770)/(2\times2580+2\times770)$
= 107.52 mm (4.23 in)

$SH = 8.2\times10^{-6}\times0.85\times193000\times$
$(1-0.00236\times107.52)\times(100-70) =$
= 30.12 MPa (4.37 ksi)

❖ Relaxation of Strands

$$RE = \left[K_{re} - J(SH + CR + ES)\right]C$$

f_{pi} = 1355.79 MPa(196.64 ksi)
f_{pi}/f_{pu} = 1355.79/1862 = 0.73
C = 0.90 (from Table 10.4.4-2)

For 270 ksi low relaxation strand, from Table 10.4.4-1:
K_{re} = 34.47 MPa (5000 psi)
J = 0.04

$RE = \left[34.47-0.04\times(30.12+0+0)\right]\times0.90 =$
= 29.94 MPa (4.34 ksi)

Hence, total stress loss is given by:
TL = 0 + 0 + 30.12 + 29.94 = 60.06 MPa (8.71 ksi)

ii. At Second Support

Over the second support the stress losses are computed as follows:

❖ Elastic Shortening

$$ES = K_{es}\left(\frac{E_s}{E_{ci}}\right)f_{cir}$$

$K_{es} = 0$ (all strands are pulled and anchored simultaneously)
Hence, $ES = 0$ MPa (0 psi)

❖ Creep of Concrete

$$CR = K_{cr}\left(\frac{E_s}{E_c}\right)(f_{cir} - f_{cds})$$

f_{cir} = 3.68 - 3.25 = 0.43 MPa (62 psi) (C)
f_{cds} = 0.33 MPa (48 psi) (T)

$CR = 1.60\times(193000/24683)\times(0.43-0.33) =$
= 1.25 MPa (0.18 ksi)

❖ Shrinkage of Concrete
Same as in the midspan
SH = 30.12 MPa (4.37 ksi)

❖ Relaxation of Strands
$RE = \left[K_{re} - J(SH + CR + ES)\right]C$
f_{pi} = 1346.55 MPa (195.30 ksi)
f_{pi}/f_{pu} = 1346.55/1862 = 0.72
C = 0.85 (from Table 10.4.4-2)

For 270 ksi low relaxation strand, from Table 10.4.4-1:
K_{re} = 34.47 MPa (5000 psi)
J = 0.04 (from Table 10.4.4-1)
RE $= \left[34.47-0.04\times(30.12+1.25+0)\right]\times0.85 =$
= 28.23 MPa (4.09 ksi)

Hence, total stress loss is

TL = 0 + 1.25 + 30.12 + 28.23 =
= 59.60 MPa (8.64 ksi)

The outcome of the stress loss calculations is summarized in Table 10.5.2-1. The loss due to elastic shortening and creep are effectively zero. For the problem at hand, at the locations considered the net strain of concrete under selfweight is tensile. Strictly speaking, there will be an increase in the tendon stress, as opposed to decrease. However, in practice, the change is assumed to be zero. This is a common case, since at most design-critical sections along a member, tendons are located in the tensile zone of the section.

TABLE 10.5.2-1 Summary of Long-Term Stress Losses at Midspan (T175)

	ksi	MPa	%
Elastic shortening (ES)	0	0	0
Creep (CR)	0.18	1.25	2
Shrinkage (SH)	4.37	30.12	51
Relaxation (RE)	4.09	28.23	47
Total	8.64	59.60	100

10.6 NOTATION

a = Anchor set;
A = cross sectional area;
CR = stress loss due to creep;
e = eccentricity of tendon from centroidal axis;
E_c = concrete's modulus of elasticity at 28 days;
E_{ci} = concrete's modulus of elasticity at stressing age;
ES = stress loss due to elastic shortening;
E_s = strand's modulus of elasticity;
f_{cds} = stress in concrete at center of gravity of tendons due to all superimposed permanent dead loads that are applied to the member after it has been prestressed;
f_{cir} = net stress in concrete at center of gravity of tendons immediately after prestress has been applied to *concrete;*
f_{cpa} = average compressive stress in concrete immediately after stressing, at a hypothetical location defined by the center of gravity of tendons;
f_{pi} = stress in tendon immediately after transfer of prestressing;
f_{pu} = ultimate strength of strand;
I = second moment of area (moment of inertia);
J = a coefficient for stress relaxation in tendon;
K = wobble coefficient of friction expressed per unit length of strand;
K_{cir} = an adjustment coefficient for loss due to elastic shortening;
K_{cr} = creep coefficient;
K_{es} = a coefficient for elastic shortening stress loss calculation;
K_{re} = a coefficient for stress relaxation in tendon;
K_{sh} = a shrinkage constant;
M = moment;
P_x = stress at distance x from the jacking point;
RE = stress loss due to relaxation of tendon;
RH = relative humidity (percent);
SH = stress loss due to shrinkage of concrete;
V/S = volume to surface ratio;

Y_b = centroidal axis to bottom fiber;
Y_t = centroidal axis to top fiber;
α = change of angle in strand (radians) from the stressing point to distance X; and
μ = coefficient of angular friction.

10.7 REFERENCES

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CHAPTER 11

STRUCTURAL MODELING OF POST-TENSIONED TENDONS

Transfer Plate under Construction
(Jabal Omar Tower; KSA; P629)

11.1 STRUCTURAL MODELING REQUIREMENTS OF PRESTRESSING TENDONS

Prestressing is used to control crack formation in concrete, reduce deflections, and add strength to the prestressed member. Prestressing thus plays a significant role in the structural integrity and desired response of a member.

The authenticity and relevance of the analysis of a prestressed concrete member rests, first and foremost, on appropriate modeling of the tendons. Several different modeling schemes are used to represent prestressing tendons, each of which has some degree of approximation [Scordelis, 1984; Ketchum et al, 1986]. This Chapter provides a brief description of each scheme and places their comparative features in perspective.

The focus of this Chapter is on the post-tensioning tendons. These are groups of prestressing strands, wires, or rods, which are stressed against the concrete member after the concrete is set. The tendons are typically given a profile in the vertical plane to enhance their load resisting characteristics.

The contribution of a tendon to the response of the prestressed member depends on the stress in the tendon at service and strength limit conditions, the tendon's profile, and its cross-sectional area. There has been little difficulty in representing the tendon profile accurately in structural analysis. The challenge facing the various modeling schemes has been the accurate determination and representation of the stress in the tendon, including immediate and long-term effects. Depending on the structure, the validity of the overall analysis may depend upon the inclusion of such effects in the analysis model.

Critical considerations in the structural modeling of post-tensioned tendons are [Aalami, 2000]:

A. Immediate Loss of Stress in a Tendon: Figure 11.1A-1a shows a post-tensioned tendon within a partially displayed concrete member. When the tendon is pulled with a force F_o at the stressing end, it will elongate. The elongation will be resisted by friction between the strand and its sheathing or duct, however. As a result of this friction, there will be a drop in the force in the tendon with distance from the jacking end. The friction is composed of two effects: curvature friction which is a function of the tendon's profile, and wobble friction which is the result of minor horizontal or vertical deviations from the specified profile.

After they are stressed, the tendons are typically anchored with conical wedges. The strand retracts upon release and pulls the wedges into the housing of the anchorage device; this forces the wedges together and locks the strand in place. The retraction of the tendon results in an additional stress loss over a short length of the tendon at the stressing end. This loss is illustrated by the difference between the jacking force and the final force at the left end of the force profile in Fig. 11.1A-1b.

Part (b) of the figure shows the variation in force along the tendon. In general, the stress loss will depend on the tendon's length, its profile, its friction characteristics, the design of its locking mechanism and the stressing force. The combined loss due to all of these effects is commonly referred to as the *friction and seating loss.* The force profile will be similar for a tendon stressed at two ends; although there will be a seating loss at each end the total loss due to friction will be less.

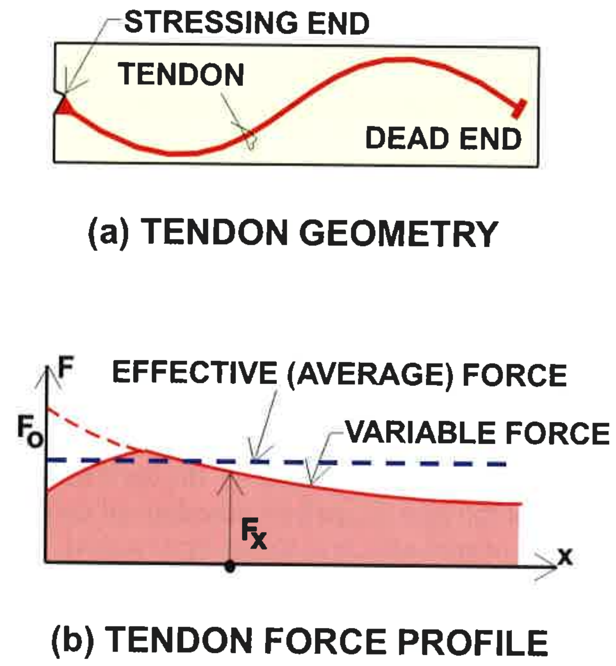


FIGURE 11.1A-1 Loss of Prestressing Due to Friction and Seating (P612)

B. Elastic shortening: Most prestressed members are reinforced with several tendons which are stressed and anchored one after another. As each tendon is stressed, the compression in the concrete member increases. The elastic shortening of the concrete due to the increase in compressive stress causes a loss of prestressing force in tendons which were previously stressed and anchored. The stress loss in each tendon will depend on the total number of tendons in the concrete member and the sequence of stressing among other factors.

C. Long-term losses: Long-term losses cause a reduction in tendon stress with time. These losses are due to several factors:

(i) Relaxation of the prestressing steel
Prestressing tendons lose a fraction of their initial stress with time due to the metallurgical characteristics of the prestressing material. The loss in stress at any location along a tendon depends on the current value and duration of the stress at that location.

(ii) Shrinkage in concrete
A significant cause of prestressing loss is shrinkage shortening of the concrete which houses the tendon. This results in a corresponding shortening of the tendon and thus a direct reduction in tendon stress. The reduced tendon stress slightly reduces the rate and amount of stress loss due to relaxation of the prestressing steel, however.

(iii) Creep in concrete
In grouted (bonded) post-tensioning systems, there is strain compatibility between the tendon and the concrete. Creep strain in the concrete adjacent to the tendon thus causes a decrease in tendon stress, if concrete adjacent to tendon is in compression. For unbonded tendons, the decrease in stress along the tendons due to creep of the concrete is generally a function of the overall (average) precompression of the concrete member.

(iv) Change in stress due to bending of the member under applied loading
As with nonprestressed steel, the bending of a concrete member due to applied (dead and live) loading results in a change in stress of the prestressing tendons. The stress change for grouted tendons is the same as that of any nonprestressed steel located at the tendon position. The stress change due to bending is usually not viewed as a stress loss since, in most cases, there is an increase in stress. For a rigorous evaluation of the affected member, however, this change in stress must be accounted for, particularly when large deflections are anticipated.

11.2 STRUCTURAL MODELING OPTIONS OF PRESTRESSING TENDONS

For structural analysis, the prestressing tendons can be modeled either as a loading applied to the hosting member or as a component which resists the applied loading in conjunction with the hosting member. Techniques which model the tendon as applied loading include load balancing, modeling through primary moments, and equivalent load through discretization of the tendon force.

11.2.1 Modeling of Tendon as Applied Loading

A. Simple Load Balancing: Load balancing, the method introduced by T. Y. Lin [1963] is the simplest and most expedient method of modeling tendons. It is the most commonly used method in building design and when it is applied judiciously and its limitations are recognized, it is a powerful technique.

In its simplest form, load balancing can be applied under the following conditions:

- ❖ The member is prismatic with no change in the position of its centroidal axis.
- ❖ The tendon profile in each span can be approximated as a single, continuous parabola.
- ❖ The change in stress along the length of the tendon is small and does not affect the analysis. In other words, an effective (average) force can be assumed for the tendon.
- ❖ The effect of axial loading due to prestressing and the flexure of the member due to prestressing are independent from one another (decoupled).

If the given conditions are met, the impact of a tendon, removed from its housing, can be approximated by uniform loads on each span, if tendon is in shape of a continuous parabola from one span support to the next, as illustrated in the example of Fig. 11.2.1A-1a. In practice, tendons change curvature over the supports and the distribution of force, depending on the shape of tendon, can be as shown in part (b) of the figure.

It is emphasized that the premise of load balancing is constant force along the length of a tendon.
The force of the tendon on the concrete is considered to balance (offset) a portion of the load on the member, hence the "load balancing" terminology. The loading from the removed tendon (Fig. 11.2.1A-1) is in self-equilibrium with the reactions it causes at the end of each tendon span. The lateral tendon force

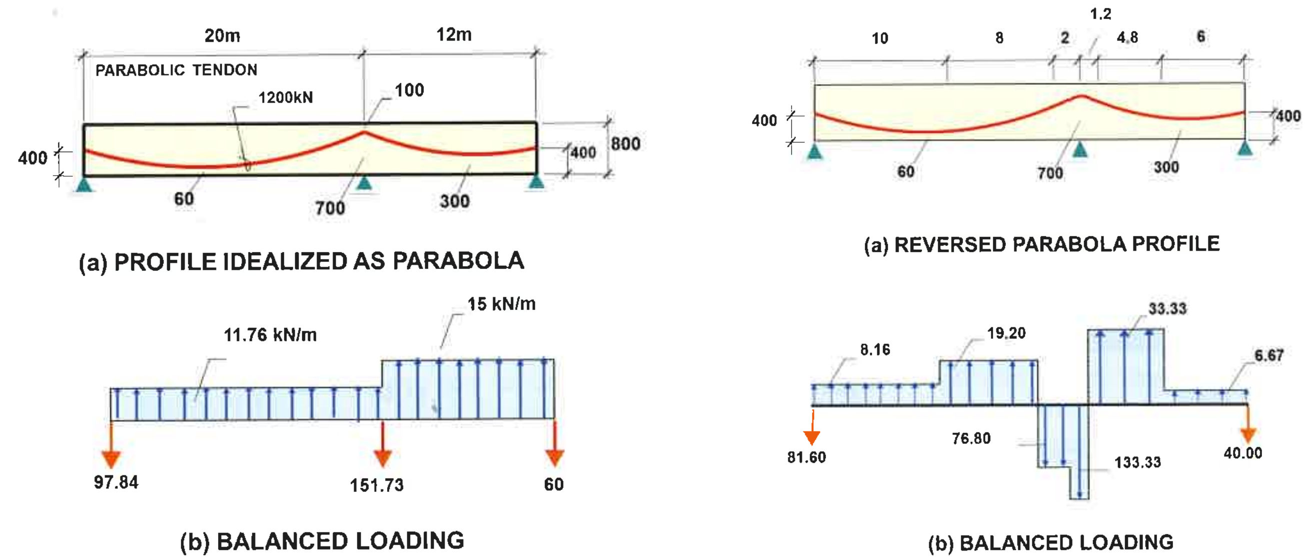


FIGURE 11.2.1A-1 Examples of Lateral Forces from Post-Tensioning Tendon (P613a,b)

and its associated concentrated loads are collectively referred to as the balanced loading. The balanced loading is independent of the support conditions of the structural member. Additional information on load balancing is provided in [Aalami, 1990].

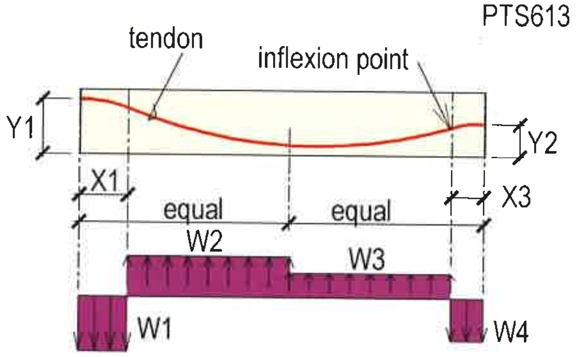
In practice, tendons cannot be placed with sharp angles over the supports, as illustrated in Fig. 11.2.1A-1 a. A gradual curve at reversal in tendon curvature is the actual condition. Tendon low points are at predetermined locations (mid-span in most building construction), and there is a gradual reversed curvature over the supports. The force imparted by a tendon to the concrete thus becomes more complex and less amenable to hand calculation. This refinement in simple load balancing is used primarily in association with automated (computer) analysis.

Reversed parabola profile is made up of basic simple parabola segments, with the forces associated with each segment. Figure 11.2.1A-2 illustrates the practical shape of a tendon with low points positioned at the center of each span and the associated lateral forces.

The principal shortcoming of the simple load balancing is that it does not apply to members whose centroidal axis changes along their length, such as members with differences in thickness, or steps. The other shortcoming is that the immediate and long-term stress losses in prestressing must be approximated and accounted for separately.

B. Extended Load Balancing: Consider the two-span beam shown in Fig. 11.2.1B-1. Because the two

PTS613



Reversed Parabola Tendon and its Balanced Loading

FIGURE 11.2.1A-2 Reversed Parabola Tendon and its Balanced Loading

spans are of different depths, the tendon ends on either end of the beam are not aligned. In order to decouple the axial and flexural actions, in accordance with the load balancing concept, it is necessary to add a moment at the point at which the centroidal axis shifts i.e. over the central support. This is shown in Fig.11.2.1B-1b and is described in more detail in Section 4.8.1C. Since this approach assumes a constant effective force, the added moment is simply the product of the post-tensioning force and the shift in the centroidal axis: 1200×0.15 = 180kNm (132.8 k-ft), where 0.15m (7.2 in) is the distance between the centroids of the two spans.

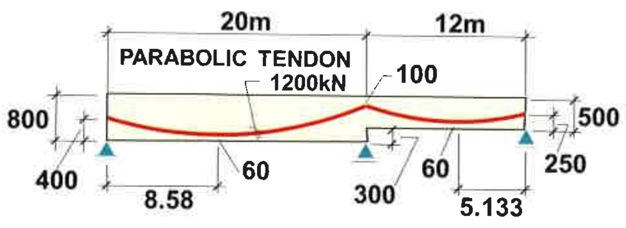
The principal advantage of the extended load balancing approach is its ability to account for nonprismatic members. It does not include a calculation of the prestressing losses.

C. Tendon Modeling Through Primary Moments: The primary moment, *Mp*, due to the prestressing force, *P*, at any location along a member is defined as the prestressing force times its eccentricity, *e*.

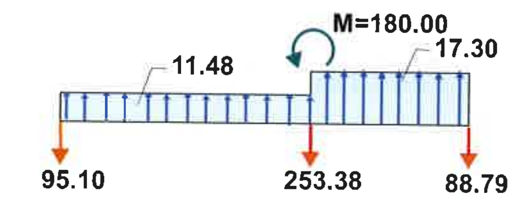
$M_p = Pe$ (Exp 11.2.1C-1)

The eccentricity of the force is the distance between the resultant of the tendon force and the centroid of the member. For the example shown in Fig11.2.1C-1, the moment at the right end of the first span is:

M_p=1200(400-100)/1000=360 kNm (265.3 k-ft)

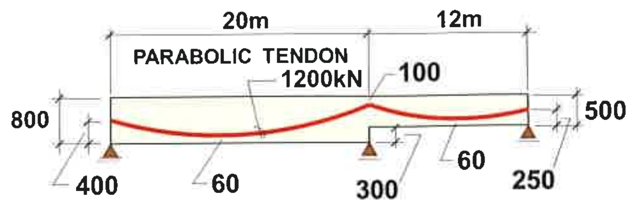


(a) MEMBER WITH SHIFT IN CENTROID

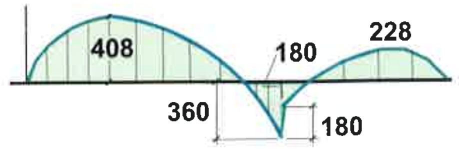


(b) BALANCED LOADING (kN, kNm)

11.2.1B-1 Extended Load Balancing (P615)



(a) MEMBER WITH SHIFT IN CENTROID



(b) PRIMARY MOMENTS (kNm)

11.2.1C-1 Modeling Through Primary Moments (P616)

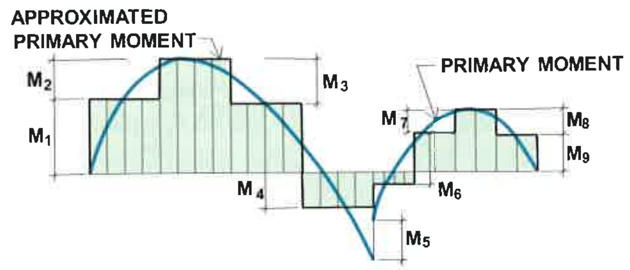
The primary moment can be used as an applied loading in lieu of the balanced loading for structural analysis. This modeling technique is more commonly used by bridge designers than building designers. It has the advantage of implicitly accounting for nonprismatic sections— a condition which is common in bridge construction. An added advantage is that by considering the primary moment at each section to be the duly adjusted force at that section times the eccentricity at the section, prestress losses along the tendon can be included. Note that if this option is adopted, the axial component of the prestress loading must be represented in its variable form to maintain equilibrium of forces.

The primary moment, whether or not is adjusted for prestress losses, depends only on the force in the tendon, the tendon profile and the cross-sectional geometry of member. It is independent of the number and location of member supports or the support conditions.

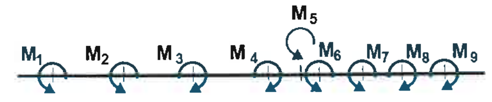
In practice, the primary moment diagram is discretized into a number of steps as illustrated schematically in Fig. 11.2.1C-2. Each discrete moment shown in part (b) of the figure is equal to the change in the value of moment between two adjacent steps in the primary moment diagram. Note that the change of moment due to the shift in the centroidal axis (M_5) is automatically accounted for.

For the example shown

M_1+M_2=408 kNm *(300.7 k-ft)*



(a) DISCRETIZATION OF PRIMARY MOMENTS



(b) APPLICATION OF PRIMARY MOMENTS AS APPLIED LOADING

11.2.1C-2 Discretization and Application of Primary Moments (P617)

D. Equivalent Force through Discretization of Tendon: In this modeling scheme, the tendon force is discretized along its length to achieve the following improvements in accuracy:

- ❖ The method accounts for the variation of force along the tendon length caused by the friction and seating of the tendon at stressing.
- ❖ The method accounts for losses in prestressing due to creep and shrinkage using an approximate procedure.

Consider a span of a continuous member and its prestressing as shown in Fig. 11.2.1D-1. The tendon length is idealized as a series of straight line segments, typically 20 segments per span. Fig. 11.2.1D-2 shows a portion of the discretized tendon. The actual distribution of prestressing force is the smooth curve marked "actual" in part (b) of the figure. For a tendon idealized as a series of straight segments there is a gradual stress loss along each segment due to wobble friction. The component of friction due to curvature (angle change), however, is concentrated at the intersection of the segments (marked as node i, i+1, ...). Hence the force distribution would be represented by a series of sloping lines with steps at the discretization points as shown in part (b). The force distribution can be further simplified by considering the force in each tendon segment to be equal to the force at the mid-point of the segments, as shown in part (c).

For a representative tendon node i, the tendon forces of the adjacent segments are F_i and F_{i-1} as shown in Fig. 11.2.1D-3. The two segment forces can be resolved into equivalent forces F_{xi} and F_{yi}, parallel and perpendicular to the centroidal axis of the hosting member. These two force components can be transferred to the centroid of the section with the addition of a moment, M_i, equal to $F_{xi}e$, where e is the eccentricity of the tendon at tendon node i. Fig. 11.2.1D-4 illustrates a series of equivalent tendon actions placed at the member centroid using this method.

This scheme, when coupled with an iterative solution strategy which involves the coupling of the tendon force and its equivalent loading, results in an analysis in which immediate prestress losses can be rigorously accounted for and long term losses can be approximated. At each iteration the prestress losses are computed on the basis of the current prestressing force. The current force is then used to compute the equivalent forces, which will, in turn, change the long-term losses. The iteration is continued until convergence is achieved.

11.2.2 Modeling of Tendon as a Load Resisting Element

Unlike the modeling schemes described in the previous sections, in this scheme the tendon is modeled as a load-resisting element; the tendon is not considered as removed from the concrete member. Rather,

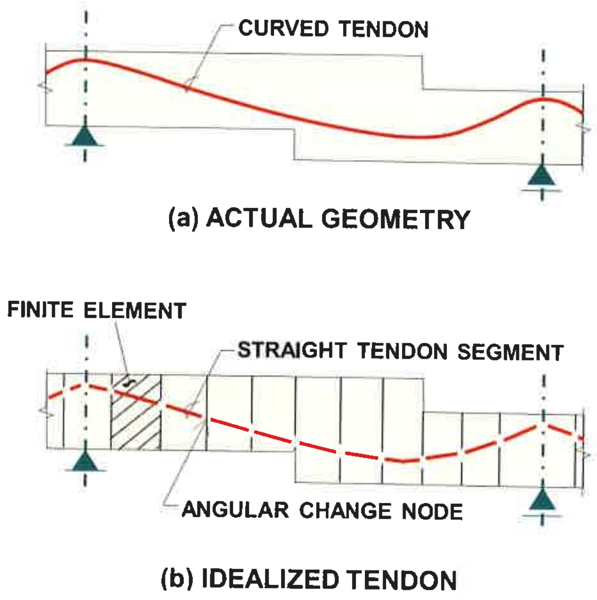


FIGURE 11.2.1D-1 Tendon Presentation by Discretization (P618)

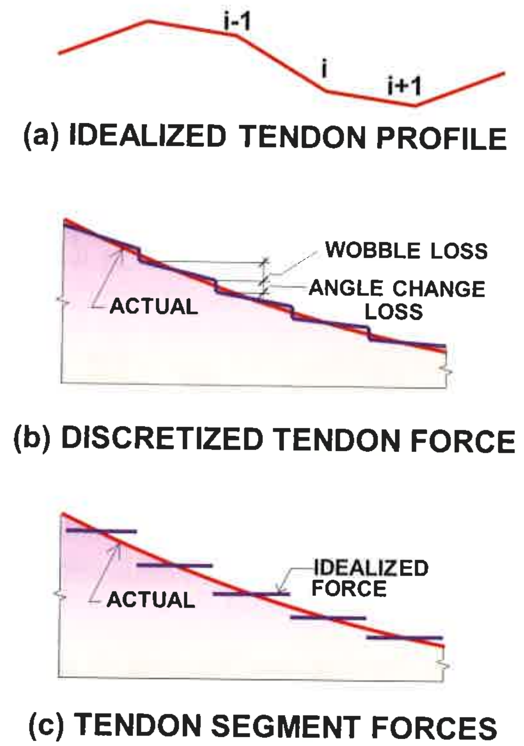


FIGURE 11.2.1D-2 Idealization of Tendon Force (P619)

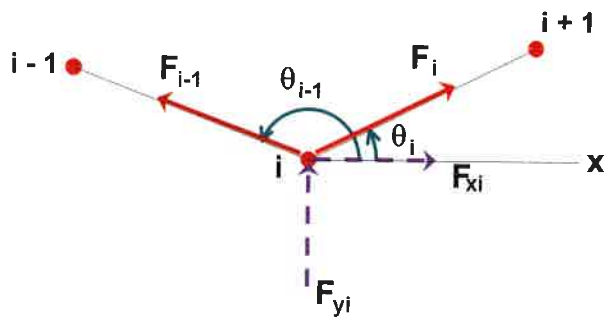


FIGURE 11.2.1D-3 Resolution of Tendon Actions at Discretization Points (P620)

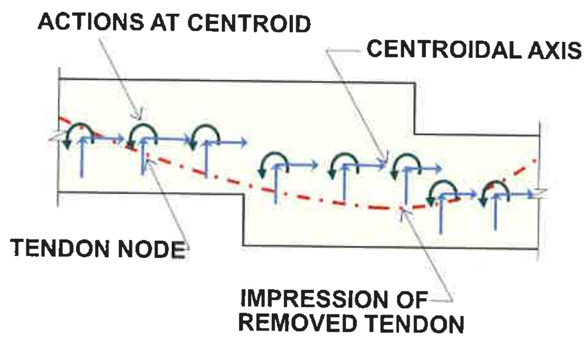


FIGURE 11.2.1D-4 Equivalent Tendon Actions at Centroidal Axes (P621)

the tendon is modeled as a distinct element linked to the concrete member. One other characteristic of this approach is that the allowance for long-term stress loss becomes an implicit feature of the computations. Separate stress loss computations are not required.

Fig. 11.2.2-1, shows a partial elevation of a post-tensioned member. The figure is used to illustrate the modeling. For simplicity, only one prestressing tendon is shown. Before describing the discrete tendon modeling option however, the equivalent load tendon modeling is reviewed.

Parts (b), (d) and (f) of the figure illustrate the representation of a tendon by means of equivalent loading. Note that the tendon is considered removed from its housing. The initial forces imparted by the tendon to the segment are transferred to the centroid of the segment (part d). These forces are considered as a constant applied loading which is not affected by creep, shrinkage, nor the deformation of the segment. Long-term loss effects are accounted for at a later stage.

In contrast, in the discrete tendon modeling scheme,

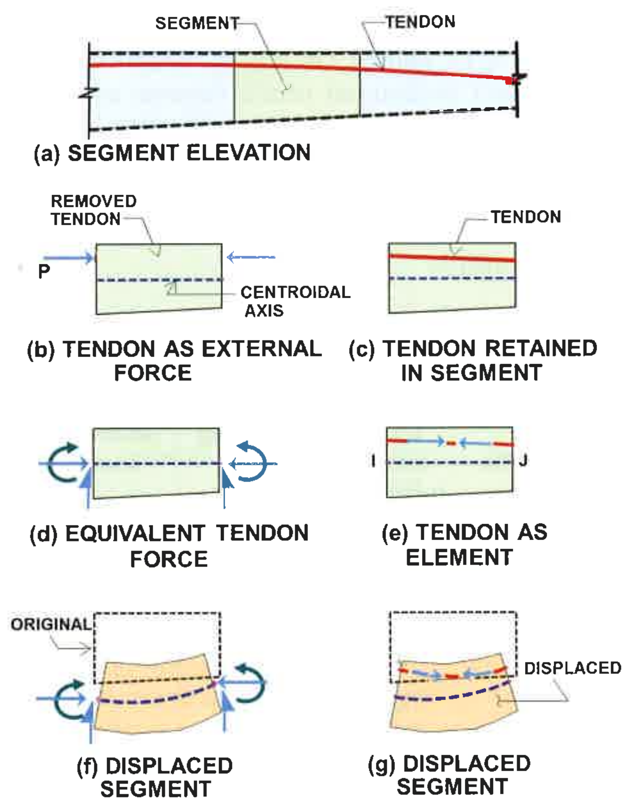


FIGURE 11.2.2-1 Discrete Tendon Modeling and Equivalent Load Modeling Options (P622)

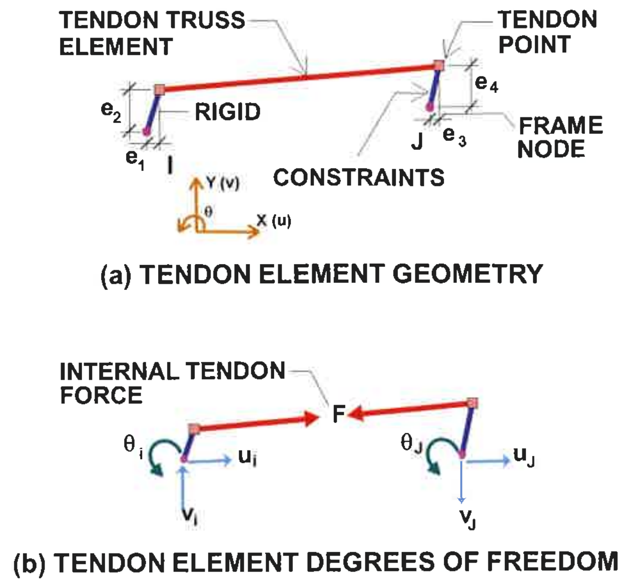


FIGURE 11.2.2-2 Prestressing Tendon Segment Idealization (P624)

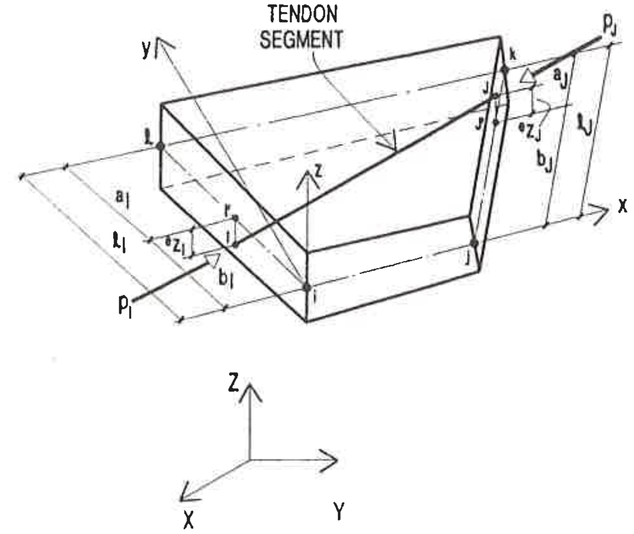


FIGURE 11.2.2-3 Representation of Segment of a Prestressing Tendon through a FEM Shell Element (P625)

the tendon is retained in position (Fig. 11.2.2-1 -c). Each tendon in the segment is viewed as an independent element, subject to displacement and change in stress based on the deformation of the segment within which it is housed, or to which it is locked (external tendons). Each tendon element is assumed to have an initial force which is determined from friction loss calculations. Any subsequent deformation of the concrete segment, such as shown in (g), will be accompanied by a compatible displacement of the tendon element, using the requirement of plane sections remaining plane (Fig. 11.2.2-2). The displacement of the tendon ends at the faces of the hosting concrete segment results in a change in tendon force.

Observe that in this modeling scheme there is an implicit interaction between the deformation of the hosting concrete segment and the force in tendon, irrespective of the cause of the deformation. As a result, it is not necessary to calculate the deformations due to creep and shrinkage separately in order to modify the tendon force. Likewise, the change in prestressing force due to relaxation is automatically accounted for in the equilibrium equations set up for the analysis of the segment.

This modeling scheme is the core of analysis software such as ADAPT-BUILDER [1] ABI. This modeling scheme is applicable to both beam, shell and plate problems such as floor slabs and bridge decks where the constituent elements of the slab can be viewed as consisting of concrete shell or stick elements with embedded prestressing tendon elements (Fig. 11.2.2-3).

11.2.3 Tendon Modeling Features and Comparison
Table 11.2.3-1 provides an overview of the features of each of the modeling schemes. Improvements in computational techniques and computing power are now allowing the traditional, approximate, long-term loss calculations to be replaced by techniques such as discrete tendon modeling which include implicit calculation of long-term losses. Although this Chapter has discussed discrete tendon modeling in conjunction with post-tensioned members, similar benefits are obtained when the technique is applied to pre-tensioned members.

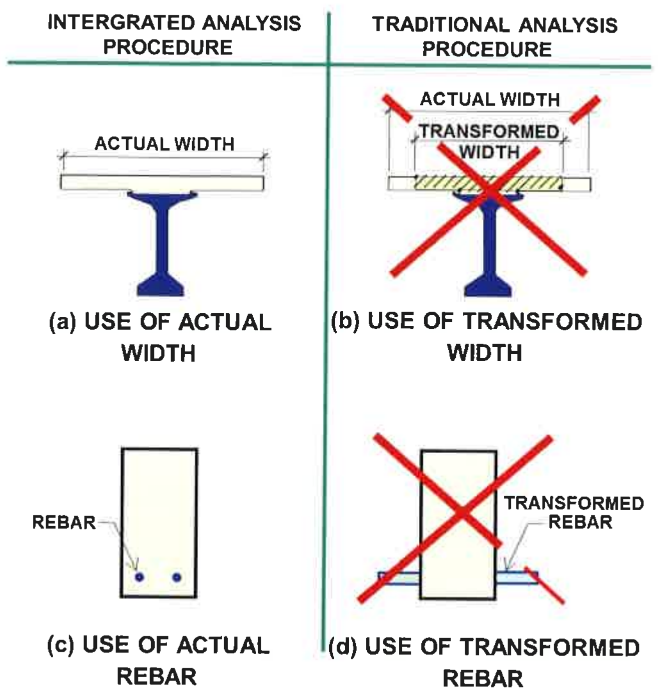


FIGURE 11.2.3-1 Comparison Between Features of Classical and "Implicit Tendon Modeling" (P626)

As an example, when using discrete tendon modeling, it no longer becomes necessary to define a "transformed width" in the analysis of precast prestressed girders provided with cast-in-place topping (Fig. 11.2.3-1a, and b). Or, it is no longer necessary to represent embedded reinforcement as a transformed equivalent concrete area (parts c and d), since in the analysis, the reinforcement is retained within its host concrete member.

11.2.4 Example

TABLE 11.2.3-1 Tendon Modeling Schemes and Features (T176)

Tendon modeling Method	**Flexure/ membrane Stresses**	**Solution mode**	**Losses at stressing included**	**Allows non-prismatic sections**	**Long-term losses included**	**Tendon modeling**	**Member deformation included**
Simple load balancing	De-coupled	Effective force	No	No	No	Applied load	No
Extended load balancing	De-coupled	Effective force	No	Yes	No	Applied load	No
Modeling through primary moments	De-coupled	Effective force	No	Yes	No	Applied load	No
Generalized equivalent load	Coupled	Variable force	Yes	Yes	No	Applied load	No
Discrete Tendon idealization	Coupled	Variable force	Yes	Yes	Yes	Resisting element	Yes

1 www.adaptsoft.com

This example is intended to illustrate the application of discrete modeling of tendons to the post-tensioned member illustrated in Fig. 11.2.1A-2.

Concrete strength	40 MPa
Concrete long-term model	ACI 209-78
Ultimate creep coefficient	2.5
Ultimate shrinkage coefficient	0.000400

Tendon area (10 strands)	988 mm2
Coefficient of angular friction	0.25 /radian
Coefficient of wobble friction	0.000066 /m
Jacking force (left end)	1470 kN
Applied sustained loading:	12 kN/m

Using a discrete tendon modeling software [ADAPT-ABI[2]], a solution for the deformation and stresses after 20 years was obtained. Each span was mod-

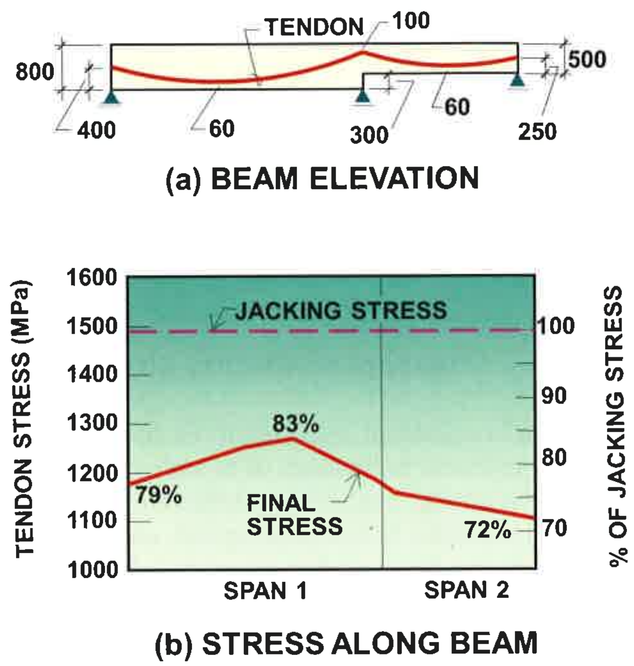


FIGURE 11.2.4-1 Structure and Stress Loss in Tendon After 20 Years (P627)

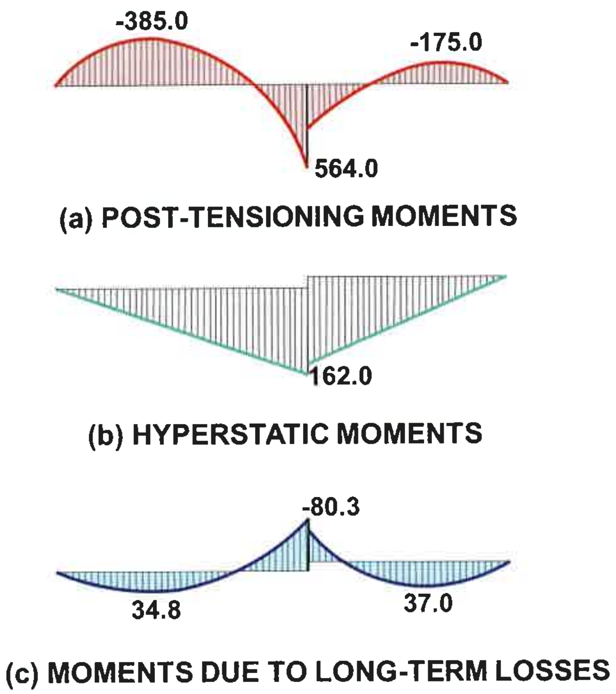


FIGURE 11.2.4-2 Post-Tensioning Actions and Losses UsingDiscrete Tendon Modeling (kN-m; 1.36k-ft) (P628)

TABLE 11.2.4-11 Tendon Modeling Schemes and Features (T177)

Tendon modeling scheme	Flexure/ membrane stresses	Solution mode	Losses at stressing included	Allows non-prismatic sections	Long-term losses included	Tendon modeling	Member deformation included
Simple load balancing	De-coupled	Effective force	No	No	No	Applied load	No
Extended load balancing	De-coupled	Effective force	No	Yes	No	Applied load	No
Modeling through primary moments	De-coupled	Effective force	No	Yes	No	Applied load	No
Generalized equivalent load	Coupled	Variable force	Yes	Yes	No	Applied load	No
Discrete Tendon idealization	Coupled	Variable force	Yes	Yes	Yes	Resisting element	Yes

Bonded Tendon

2 www.adaptsoft.com

eled as 10 segments. The post-tensioning moments, hyperstatic moments and the moments due to creep and shrinkage of the beam are shown in Fig. 11.2.4-2. T change in post-tensioning moment in the second span due to creep and shrinkage is an increase from 514.20 to 564.00 kNm. The hyperstatic moment over the interior support (162 kNm) is about 30% of the post-tensioning moment (564 kNm).

The drop in tendon force after twenty years relative to the jacking force is illustrated in Fig. 11.2.4-1b.

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CHAPTER 12

SECTION DESIGN FOR BENDING

Bangkok Government Center, Post-Tensioned with Flat Duct Bonded Tendons (Courtesy PBL; P690)

12.1 BENDING DESIGN OVERVIEW

The last, but a critical stage in design of a post-tensioned member is to determine whether the available post-tensioning provides adequate capacity to resist the design moment at ultimate limit state (ULS). And, if it does not, how much non-prestressed steel should be added to the section to meet the demand. In addition, the section must be checked to have adequate ductility for a pronounced post-elastic deformation. In the general case, a typical design section of a floor system is subject to six components of actions, namely three forces and three moments as outlined in Section 4.9.5.5 At ultimate limit state (ULS), however, the design for "bending" accounts for the combined effects of the moment about an axis parallel to the plane of the slab and the axial force on the section. The foregoing moment and the axial force are handled together, separate from shear and other actions. Further, for floor system design, it is assumed that the displacement of the section is normal to the axis of the moment – normal to the plane of slab. Since "design sections" in floor systems are generally contained by adjoining parts of the floor, the assumption of displacement normal to the axis of moment is justified. There will be no "unsymmetrical" bending.

There is a fundamental difference in approach for strength design between conventionally reinforced and post-tensioned members. In practice, the strength design of a member follows its satisfactory design for in-service condition (SLS), when the location and amount of prestressing is decided. Other requirements of SLS, such as crack control, or minimum reinforcement can result in addition of non-prestressed reinforcement to post-tensioning tendons. Figure 12.1-1a illustrates a post-tensioned member with a continuous tendon, along with the positive and negative design moment capacity that the tendon provides. Part (c) of the figure illustrates the additional capacity that will be available from non-prestressed top reinforcement that may be required to complete the serviceability requirements. When designing according to ACI 318, due to stricter requirements on crack control for two-way systems, the post-tensioning required for service condition, generally provides adequate capacity for the strength requirements of the same member. Mostly, no added non-prestressed steel will be required at ULS. Consequently, the process starts by first finding the design capacity of a section. And, if the capacity is not adequate, compute the amount of supplemental non-prestressed reinforcement.

This Section reviews the procedures for capacity check, and design of sections under the combined actions of bending and axial forces. Following a brief introduction of the basics for the general procedure, the Chapter details the ACI 318 and EC2 recommendations for design of sections in bending. Each code-based procedure is followed by a numerical example.

Figure 12.1-1a is a member post-tensioned with a continuous tendon. The tendon provides a base moment capacity as shown in part (b) of the figure. Where added reinforcement over the supports, and possibly bottom rebar in the field are required to meet the serviceability requirements, the base capacity of the member will be enhanced as shown in part (c) for the condition of added top rebar.
Another aspect of design in practice is that the de-

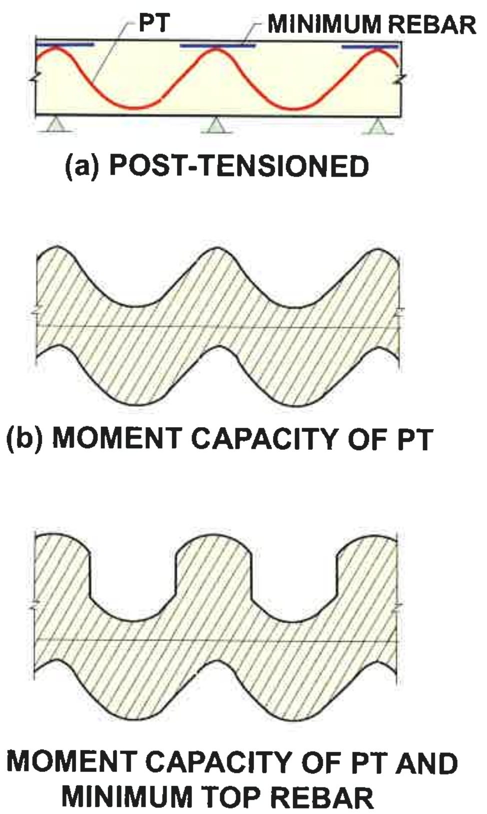

FIGURE 12.1-1 Base Moment Capacity of a Post-Tensioned Member (P515)

sign sections are not always rectangular or in shape of a simple T, reinforced with only one layer of prestressing tendons and another layer of rebar. In the general case, the geometry of a design section can be non-standard, due to steps in a floor or other irregularities. Further, for design sections that extend over the entire design strip, there are generally multiple tendons crossing the section at different locations and at different angles. Similarly the nonprestressed steel is likely to be multi-layered and not necessarily normal to the section. This is particularly true, where the design engineer specifies a mesh as base reinforcement throughout a floor system, and wishes to account for the availability of the base reinforcement in the strength design of the section. It follows that for practical design of a floor system, the section design for strength should be general, and capable to account for the complexity in geometry and layout of reinforcement. This is particularly true, where software is used to process the design automatically. In such a case, the idealizations or simplifications that are generally followed in hand calculations are not available.

12.2 DESIGN BASED ON STRAIN COMPATIBILITY

The basic premise of strength design of a section subject to bending and axial load is that the deformation follows the premise of "plane sections remain plane." In addition, when dealing with design sections cut from the continuum of a floor system, it is assumed that the section undergoes uniaxial bending and deflects normal to the slab surface, irrespective of whether the section and/or its reinforcement is symmetrical about an axis normal to the slab surface or not. The argument is supported by the fact that, when in place, each design section is bound by the constraint against lateral displacement and rotation by the adjoining parts of the construction.

The strain compatibility and section design concept are outlined by way of an example of a simple rectangle described next:

Figure 12.2-1 shows the geometry, strain and force distribution for a rectangular section reinforced with prestressing and non-prestressed steel. T_p, T_s, C_s and C_c represent forces from prestressing, non-prestressed steel, compression steel and compression of concrete respectively. At strength limit state, these forces combine to provide the design capacity of the section. The combined effects of the forces follow the relationships given below.

$$C - T = N \qquad \text{(Exp 12.2-1)}$$

Where,

C = Total compression on the section;
T = total tension on the section; and
N = externally applied axial force.

The force in each component of the section is based on its computed strain from the linear distribution of strain on the section, and component's stress-strain relationship. Figure 12.2-2 shows the simplified versions of the stress-strain relationships commonly

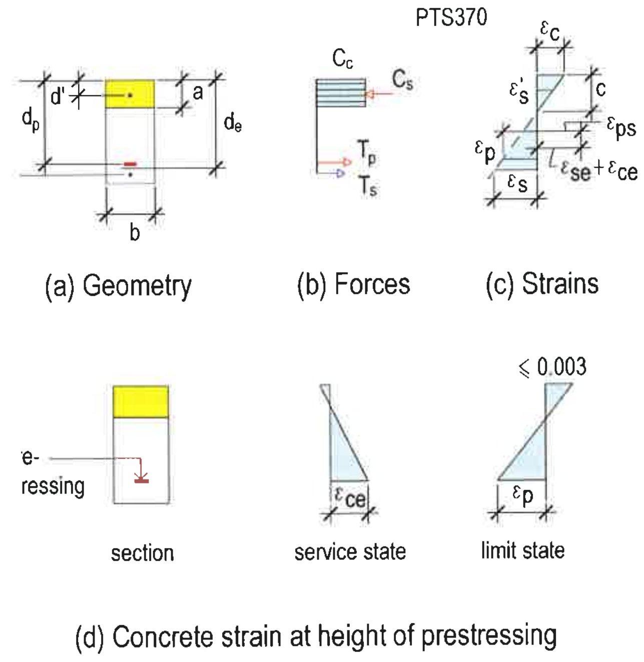

FIGURE 12.2-1 Forces and Strains

assumed. A number of commercially available software allow for strain hardening in the stress-strain relationship of the steel. The stress-strain relationship shown in part (c) of the figure applies to bonded tendons. Unbonded tendons do not follow the local strain in concrete, at ultimate state of a section, the force developed in an unbonded tendon is based on empirical values, or relationships recommended in the building code used. This is further discussed in Section 12.3

Where a base reinforcement or tendon is not normal to the section, its force component normal to the section is considered in the preceding equilibrium relationships.

When calculating the capacity based on building code requirements, additional conditions are imposed. The general procedure followed is: (i) assume a strain distribution over the section; (ii) calculate the associated stresses in each of the constituent parts; (iii) calculate the force contribution of each component; (iv) check the outcome against the two equilibrium equations. If the equations are not satisfied, the strain distribution is adjusted for a new trial. In the process, the assumed distribution of strain at each iteration is checked to be less than the maximum values permitted in the respective code.

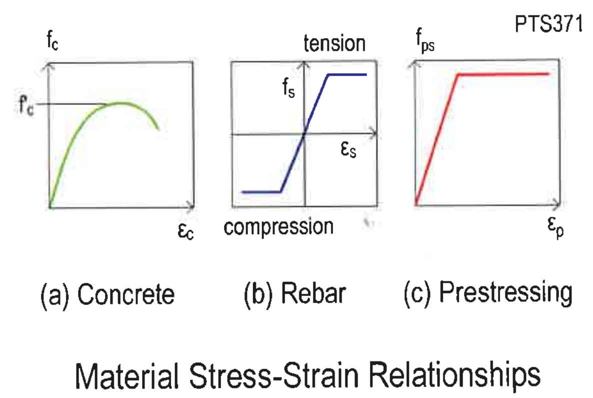

FIGURE 12.2-2 Material Stress-Strain Relationship

ACI 318 sets the maximum concrete strain in compression at 0.003. EC2 recommends 0.0035 for the same strain. Further, each code reduces the force values calculated for each component by a certain fraction deemed to account for the uncertainties in material properties and the reliability of computational process.

In addition to the limitations noted in the foregoing, the codes impose another restriction in design to ensure that at ultimate limit state, the failure will be ductile. The ductility of a section is deemed adequate, if failure is initiated by yielding of reinforcement prior to collapse. This is achieved by the restriction on the maximum compressive strain in concrete (ε_{cmax}) and the maximum allowable depth of compression zone (c_{max}). This is shown in Fig. 12.2-3 for ACI code.

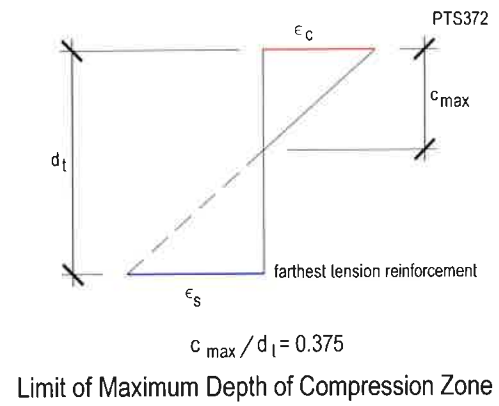

FIGURE 12.2-3

In regards to the maximum depth of neutral axis, the codes covered herein have the following suggestions:

A. ACI 318 Ductility Requirement[1]: ACI 318 sets the maximum depth of compression zone at $0.375d_t$, where dt is the distance from extreme compression fiber to the farthest reinforcement in the section. The restriction can be relaxed, if the strength reduc-

1 ACI 318-11 Section R9.3.2.2

tion factor (Φ) is reduced. The strength reduction factor is a coefficient that controls the safety factor of a design. For the condition under discussion the following governs:

$$M_u = \Phi M_n \quad \text{(Exp 12.2A-1)}$$

Where,

M_u = design moment capacity;
M_n = nominal capacity obtained from the linear strain distribution; and
Φ = strength reduction factor, recommended as 0.90 for bending, but adjustable based on Fig. 12.2A 1

ACI permits exceeding c_{max} by lowering the strength reduction factor Φ as shown in Fig. 12.2A 1, thus enabling a continuous transition in design from pure bending to pure axial loads. However, this provision does not pertain to design of slabs in common building structures, since the floor slabs are primarily acted upon in bending.

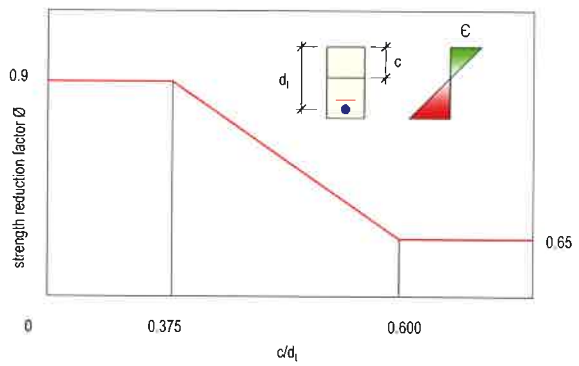

FIGURE 12.2A-1

B. EC2 Ductility Requirements[2]: The maximum allowable value for the depth of compression zone (x) is determined based on concrete strength of the section f'_c [f_{ck}].Code provides two values based on stress-strain diagram used whether parabola-rectangle or bi-linear.

For parabola- rectangle stress-strain diagram

- For $f_{ck} \leq$ 50MPa (7252 psi),
  x =0.43h (Exp 12.2B-1)
- For f_{ck} > 50MPa (7252 psi),
  $x = (1\text{-}\varepsilon_{c2}/\ \varepsilon_{cu2})h$ (Exp 12.2B-2)

[2] EC2(EN 1992-1-1:2004(E)), Section 6.1

For bi-linear stress diagram

- For $f_{ck} \leq$ 50MPa (7252 psi),
  x = 0.5h (Exp 12.2B-3)
- For f_{ck} > 50MPa (7252 psi),
  $x = (1\text{-}\varepsilon_{c3}/\ \varepsilon_{cu3})h$ (Exp 12.2B-4)

The values of ε_{c2}, ε_{cu2}, ε_{c3} and ε_{cu3} are according to Table 3.1 of EC2.

C. Design Conditions Accounting for Ductility: Since in prestressed members, there exists a given amount of prestressing, and possibly additional non-prestressed reinforcement, in regards to compliance with the ductility requirements, six design conditions as outlined in Fig. 12.2.C-1 arise. Depending on the design condition, none, tension, compression, or both tension and compression reinforcement may be required.

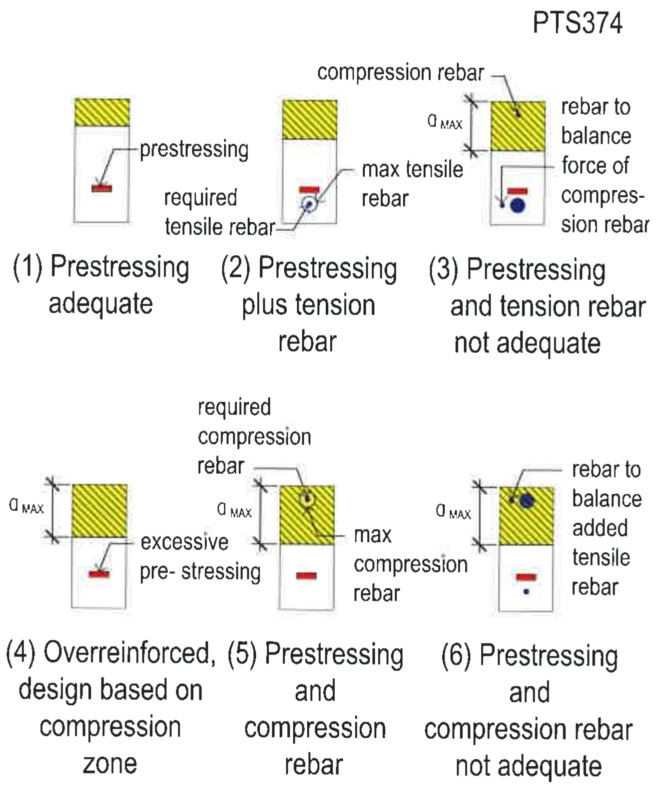

FIGURE 12.2C-1 Design Conditions

The conditions are outlined below briefly.

Condition 1: This is the case where the capacity provided by the available prestressing is in excess of that required to resist the design moment M_u. Further, the associated depth of the neutral axis is less than the allowable limit ($c < c_{max}$). This is the most common situation. The section is adequate. No additional reinforcement is required.

Condition 2: The available prestressing is not adequate to resist the design moment. Added rebar in amount A_s is required to supplement the prestressing A_{ps}. The combined areas of A_{ps} and A_s result in c < c_{max}. The larger circle shown in the figure around the rebar symbolically represents the maximum area of rebar ($A_{s\,max}$) that would bring the section to its ductility threshold of ($c = c_{max}$).

Condition 3: Prestressing and non-prestressed tension reinforcement in the amount that will result in $c = c_{max}$ are not adequate to resist the demand moment. In this case, the neutral axis is set at c_{max}, compression and tension reinforcement is added in an amount and at the location that will retain the position of the neutral axis at c_{max} and provide adequate capacity to meet the demand moment M_u.

In other words, the shortfall in design capacity beyond that available through the maximum value of compression force associated with c_{max} must be generated by a force couple resulting from addition of tension and compression rebar. Depending on the position of compression rebar, it may not reach its yield stress at the section's ultimate limit state.

Condition 4: In this case, the amount of available prestressing in the section is excessive, in the sense that at ultimate limit state the tensile force of the existing prestressing will result in a compression zone that exceeds the maximum depth set for ductility ($c > c_{max}$). An example for such a condition is in Fig. 12.2C-2. In the beam shown, the post-tensioning is determined based on the mid-span moment, but it is continued to the end supports. At regions near the end supports, the smaller effective depth of the section leads to a smaller c_{max} than that available at midspan.

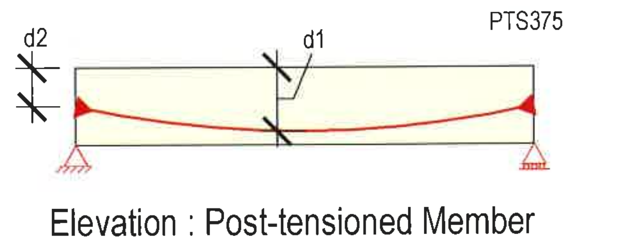

FIGURE 12.2C-2

In this condition, the design capacity of the section is based on the capacity of the section's compression zone. The section capacity based on the force generated in the compression zone is calculated setting the neutral axis at c_{max}.

Condition 5: Where the force from prestressing alone will result in $c > c_{max}$, and the capacity based on the compression zone with c set at c_{max} is not adequate to provide adequate resistance, compression rebar is added. The design capacity is based on the combined contribution of the force provided by concrete in the compression zone and the added reinforcement placed in the compression zone, using the distance to the tendons' centroid as lever arm for the compression rebar. Note that for this condition, the equilibrium in the format stated in Section 12.2 for the resisting and demand moments will neither be satisfied, nor is it necessary to complete the design.

Condition 6: In this case, the demand moment is in excess of the resistance that can be provided by assuming the maximum allowable depth of the compression zone and the maximum compression rebar that will be in equilibrium with the tensile force provided by post-tensioning. The overage of demand moment will then be resisted by a couple that will be generated through addition of both tension and compression rebar. The location of added rebar along with the maximum depth of compression zone will be used to determine the force in the components of the shortfall couple.

D. Strains in Post-Tensioned Tendons and Strain Compatibility: Using a linear distribution of strain over a design section, the strain in each of the section's reinforcement is defined by the ordinate of the strain diagram associated with the position of the reinforcement. For example, in Fig. 12.2D-1 the strain ϵ_s is for the non-prestressed reinforcement A_s. The same is not true for prestressed steel, be it pre- or post-tensioned. In addition to the strain shown by the diagram in Fig. 12.2D-1, there are strain com-

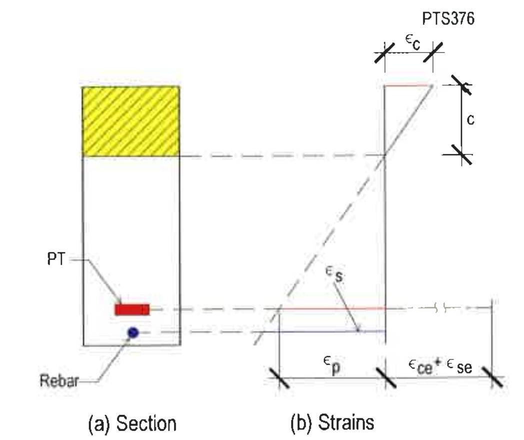

FIGURE 12.2D-1 Distribution of Strain at Ultimate Limit State

ponents from prestressing. The following describes the parts that contribute to the total strain in prestressing steel, and how to determine them. It is important to note that pre-tensioned steel and grouted tendons are somewhat similar, since they bond to concrete that contains them. Once the bond is established, there is compatibility of strain between the prestressing steel and the concrete that encases it. Unbonded tendons, on the other hand, do not follow the strain compatibility associated with grouted tendons. Their strain at ULS is based on experimental data, using empirical relationships given in respective building codes. The following explains the strain at ULS for grouted tendons.

Refer to Fig. 12.2D-1. The strain in prestressing tendons is given by

$$\varepsilon_{ps} = \varepsilon_{se} + \varepsilon_{ce} + \varepsilon_p \qquad \text{(Exp 12.2D-1)}$$

Where,

- ε_{ps} = strain in prestressing steel at strength limit state, that is when section develops its nom inal moment capacity;
- ε_{se} = strain in prestressing steel due to effective stress in tendon after all losses have taken place (zero dead and live load assumption);
- ε_p = strain in concrete at level of prestressing steel centroid when section develops its nominal strength; and
- ε_{ce} = concrete strain at level of tendon centroid, after all tendon losses have taken place and no load on the structure. This strain is also referred to as decompression strain.

The determination of the location of the neutral axis "c" for the general case—where the section has multiple layers of reinforcement—is done by trial and error, using the following procedure.

i. Assume concrete strain at the compression fiber to be the code specified value of 0.003 or 0.0035; depending on the code used;
ii. select a trial value for the depth of the neutral axis "c";
iii. using the value of "c" construct a linear distribution of strain through the depth of the section passing through the previous two points;
iv. using the assumed strain distribution, determine the stress of each rebar, or the prestressing steel using the procedure described below;
v. calculate the force in the compression zone and the tensile reinforcement;
vi. apply the material factor to each of the calculated forces if so required by the respective building code; and
vii. check the equilibrium of section (T=C). If the equilibrium is not satisfied, modify "c" and repeat the above steps, until the correct value of "c" is determined.

The following numerical example helps to explain the significance of each strain component.

EXAMPLE 1: ACI 318-Strain Compatibility Design Example for Bending

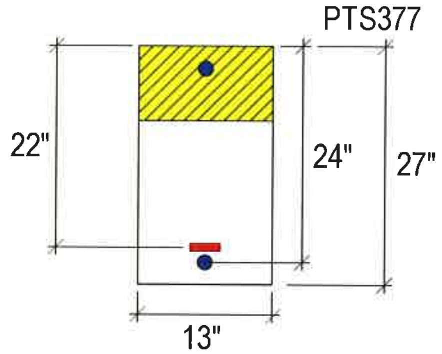

FIGURE EX1-1 Geometry of Section

Using strain compatibility, determine the moment design capacity (ϕM_n) of the section shown:

Given:
Geometry:
Depth (h) = 27 in (686 mm)
Stem width (b) = 13 in (330 mm)
Distance to tendon (d_p) = 22 in (556 mm)
Distance to rebar ($d_r = d_t$)= 24 in (610 mm)
Concrete:
f'_c = 4000 psi (27.58 MPa)
E_c = 3605 ksi (24,856 MPa)
Prestressing:
3 - ½ in strand; A_{ps} = 3 x 0.153 = 0.459 (296 mm²)
Grouted system
f_{pu} = 270 ksi (1860 MPa)
E_{ps} = 28000 ksi (193,050 MPa)
Effective stress, f_{se} =175 ksi(effective stress after all losses) (1,206 MPa)
Rebar:
f_y = 60 ksi (413.69 MPa)
E_s = 29000 ksi (199,945 MPa)
1 # 6; A_S = 0.44 in² (284 mm²)
Strength reduction factor $\Phi = 0.90$

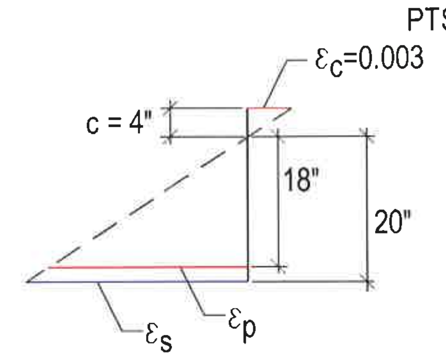

FIGURE EX1-2 Distribution of Strain at Strength Limit State

Required:
Design moment capacity ΦM_n

1 - Determine the depth of neutral axis
Using trial and error, the depth of neutral axis "c" that satisfies the equilibrium of forces (T=C) is determine to be 4 in. (102 mm). The distribution of strain over the section, based on maximum concrete strain (0.003) and 4 in (102 mm) depth of the neutral axis is shown in Fig. EX1-2.

2 - Determine the strains

2.1 Rebar

ε_s = (20/4) x 0.003 = 0.015 > (f_y/E_s) = 60/29000 = 0.00207

Hence, rebar yields

2.2 Prestressing

$\varepsilon_{ps} = \varepsilon_{se} + \varepsilon_{ce} + \varepsilon_p$

$\varepsilon_{se} = f_{se} / E_{ps}$= 175/28000 = 0.00625

$$\varepsilon_{ce} = F_{pt}\left(\frac{1}{A}+\frac{e^2}{I}\right)/E_c$$

Where,

A = gross area of cross section;
e = eccentricity of tendon with respect to the centroid of gross cross section; and
I = moment of inertia of section.
A = 13 x 27 = 351 in² (226451 mm²)

$$I = \frac{13\times 27^3}{12} = 21323\text{in}^4\left(8.875\text{e}{+}09\ \text{mm}^4\right)$$

F_{pt} = 175 x 0.459 = 80.33 k (357.32 kN)

e = 22 - 27/2 = 8.5 in (216 mm)

$$\varepsilon_{ce} = 80.33\left(\frac{1}{351}+\frac{8.5^2}{21323}\right)/3605 = 1.39\times 10^{-4}$$

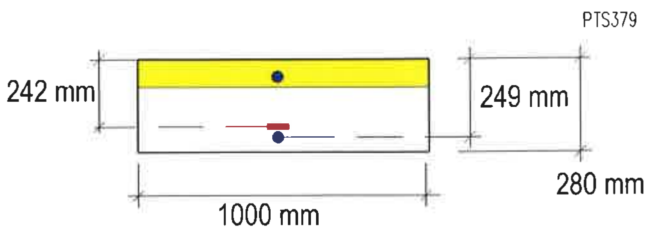

FIGURE EX2-1 Geometry of Section

$\varepsilon_p = (18/4)\times 0.003 = 0.0135$
ε_{ps} = 0.00625 + 0.0135 + 0.000139 = 0.0199
$f_{ps} = \varepsilon_{ps}$ x E_{ps}= 0.0199 28000 > 270 ksi
use 270 ksi (1860 MPa)

3 - Determine the forces
$a = \beta_1 c$ = 0.85 x 4 = 3.4 in (86 mm)
C_c = 0.85 x 4 x 13 x 3.4 = 150.28 k (668.48 kN)
T_s = 0.44 x 60 = 26.4 k (117.43 kN)
T_{ps} = 3 x 0.153 x 270 = 123.93 k (551.27 kN)

4 - Check equilibrium
$C = C_c$ = 150.28 k (668.48 kN)
$T = T_s + T_{ps}$ = 26.4 + 123.93 = 150.33 k (668.70 kN)
$C = T$ OK

5 - Calculate design capacity
c/d_t = 4/24 = 0.167 < 0.375, hence Φ = 0.9
Take moments about the centroid of compression block
ΦM_n =
$0.9\times\left[123.93(22-3.4/2)+26.40(24-3.4/2)\right]$ = 2794.05 k-in (315.68 kNm)
ΦM_n = 2794.05/12 = 232.84 k-ft (315.68 kNm)

EXAMPLE 2: EC2 Strain Compatibility Design Example for Bending
Using strain compatibility, determine the design capacity (ΦM_n) of the section shown using EC2[3] : The section represents one meter strip of a floor slab construction (Fig. EX2-1).

Given:
Geometry:
Depth (h) = 280 mm (11 in.)

[3] EC2(EN 1992-1-1:2004(E)), Section 6.1

Width (b) = 1000 mm (39.37 in.)
Distance to tendon (d_p) = 242 mm (9.53 in.)
Distance to rebar ($d_r = d_t$) = 249 mm (9.80 in.)
Distance to compression rebar (d')=33 mm (1.30 in.)
Concrete:
f'_c (28 day cylinder) = 30 MPa (4,351 psi)
f_{cd} (concrete design stress)= 30/1.5 = 20 MPa (2,901psi)
Modulus of elasticity, E_c = 32835 MPa (4,762 ksi)
Prestressing:
3 - 13 mm strand; A_{ps} = 3 98 = 294 mm^2 (0.46 in^2)
Grouted system
f_{pu} = 1,860 MPa (270 ksi)
E_{ps} = 200,000 MPa (29,008 ksi)
Effective stress, f_{se} = 1,200 MPa (174 ksi) (after all losses)
Material factor, γ_m = 1.15
Rebar:
f_y = 460 MPa (66.72 ksi)
E_s = 200,000 MPa (29,008 ksi)
A_s (bottom) = 4 - 12 mm bars = 4 x 113 = 452 mm^2 (0.70 in^2)
A'_s (top) = 2 - 16 mm bars = 2 x 201 = 402 mm^2 (0.62 in^2)
γ_m = 1.15

Required:
Moment capacity M_n

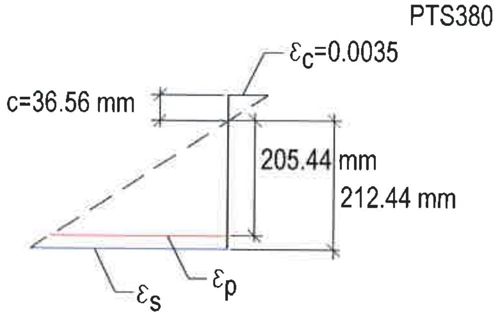

FIGURE EX 2-2 Distribution of Strain at Strength Limit State (ULS)

Using the procedure described earlier, the depth of the neutral axis "c" that satisfies the equilibrium of forces ($T = C$) is determine to be 36.56 mm (1.44 in). The distribution of strain over the section based on maximum concrete strain (0.0035) and 36.56 mm depth of the neutral axis is shown in Fig. EX-2-2

.

2 - Check rebar stresses

$$\varepsilon_s = \left(\frac{249-36.56}{36.56}\right)\times 0.0035 = 0.020$$

$$> \frac{f_y}{E_s} = \frac{460}{200000} = 0.0023$$

Hence, rebar yields.

$$\varepsilon'_s = \left(\frac{36.56-33}{36.56}\right)\times 0.0035 = 3.408 \text{ x}10^{-4}$$

$$< \frac{f_y}{E_s} = \frac{460}{200000} = 0.0023$$

Hence, compression rebar does not yield.

f'_s = 3.408 E-4 200000 = 68.16 MPa (9.89 ksi)

3 - Check prestressing values

$\varepsilon_{ps} = \varepsilon_{se} + \varepsilon_{ce} + \varepsilon_p$
$\varepsilon_{se} = f_{se} / E_{ps}$ = 1200/200000 = 0.006

$$\varepsilon_{ce} = F_{pt}\left(\frac{1}{A}+\frac{e^2}{I}\right)/E_c$$

Where,

A = gross area of cross section;
e = eccentricity of tendon with respect to the centroid of gross cross-section; and
I = second moment of area of section.
A = 1000 x 280 = 280000 mm^2 (434 in^2)
I = 1000 x 280^3 /12 = 18.293e + 08 mm^4 (2.835 x 10^6 in^4)
F_{pt} = 1200 x 98 = 118 kN per strand (26.53 k)
e = 242- 280/2 = 102 mm (4.02 in)

$$\varepsilon_{ce} = 3\times 118\times 1000\left(\frac{1}{280000}+\frac{102^2}{18.293\times 10^8}\right)/32835$$

-9.98 x 10^{-5}

$$\varepsilon_p = \left(\frac{242-36.56}{36.56}\right)\times 0.0035 = 0.0197$$

ε_{ps} = 0.0060 + 0.0197 + 0.0000998 = 0.026
f_{ps} = ε_{ps} x E_{ps} = 0.026 x 200000 = 5200 MPa > 0.9 x 1860 MPa;

use 1,674 MPa (242,800 ksi)

4 - Determine force values
α = 0.80 x 36.56 = 29.25 mm (1.15 in)
C_c = (1 x 20 x 29.25 x 1000)/1000 = 584.96 kN (131.50 k)
C_s = [402 x (68.16/1.15)]/1000 = 23.83 kN (5.36 k)
T_s = [452 x (460/1.15)]/1000
= 180.80 kN (40.65 k)
T_{ps} = [3 98 (1674/1.15)]/1000= 427.96 kN (96.21 k)

5 - Check equilibrium
$C = C_c + C_s$ = 584.96 + 23.83 = 608.79 kN (136.86 k)
$T = T_s + T_{ps}$ = 180.80 + 427.96 = 608.76 kN (136.86 k)

$C = T$ OK

6 - Calculate design capacity
c/h = 36.56/280 = 0.130 < 0.43
Take moments about the centroid of compression rebar
M_n = [427.96 (242 -33) + 180.80 (249 - 33) + 584.96 (33-29.25/2)]
M_n = 139.25 kNm (102.70 k-ft)

12.3 BENDING DESIGN BASED ON SIMPLIFIED CODE FORMULAS

For rectangular and T-sections with simple arrangement of reinforcement, pre-tensioned, or post-tensioned with either bonded or unbonded tendons (Fig. 12.3-1), major building codes suggest simplified formulas to calculate the design capacity in bending. The simplification presented in the code applies to the determination of stress in prestressing steel at ultimate limit state. The stress in non-prestressed reinforcement and in concrete, as well as distribution of linear strain over the section, maximum concrete strain (ε_c) and the maximum depth of compression zone (c_{max}) are the same as discussed in the preceding for the case of strain compatibility.

12.3.1 ACI 318 Simplified Bending Design[4]

As an alternative to a more accurate determination

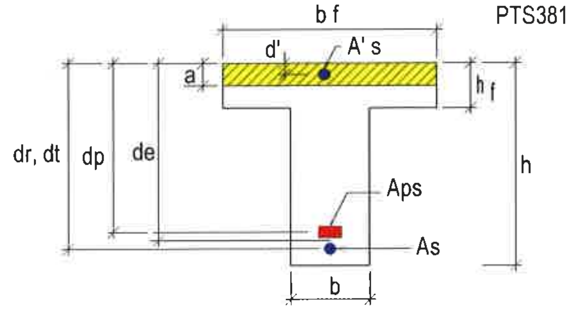

FIGURE 12.3-1 Geometry and Reinforcement

of stress in prestressing steel at ultimate limit state (f_{ps}) based on strain compatibility, the following approximate values for f_{ps} shall be permitted to be used, if the effective stress in prestressing steel (f_{se}) is not less than 0.5 f_{pu} (where f_{pu} is the specified tensile strength of prestressing steel).

A numerical example for the application of the ACI-318 simplified relationship is given in Chapter 6.

A- For Members with Bonded (Grouted) Tendons:

$$f_{ps} = f_{pu}\left\{1-\frac{\gamma_p}{\beta_1}\left[\rho_p\frac{f_{pu}}{f'_c}+\frac{d}{d_p}(\omega-\omega')\right]\right\}$$

(Exp 12.3.1A-1; US and SI)

If any compression reinforcement is taken into account when calculating f_{ps}, the term in square brackets [ ] shall not be taken less than 0.17 and the distance from the extreme compression fiber to the centroid of compression reinforcement shall not be greater than 0.15d_p.

B- For Members with Unbonded Tendons:
(i) For span-to-depth ratio of 35 or less.

$$f_{ps} = f_{se} + 10{,}000 + \frac{f'_c}{100\rho_p} \qquad \text{(Exp 12.3.1B-1 US)}$$

In the above relationship f_{ps} shall not be greater than f_{py}, nor (f_{se} + 60,000).
In SI system of units the associated relationship is:

$$f_{ps} = f_{se} + 70 + \frac{f'_c}{100\rho_p} \qquad \text{(Exp 12.3.1B-2 SI)}$$

Where, f_{ps} shall not be greater than f_{py}, nor (f_{se} + 420).
(ii) For members reinforced with unbonded tendons and span-to-depth ratio greater than 35

[4] ACI 318-11 Section 18.7

$$f_{ps} = f_{se} + 10{,}000 + \frac{f'_c}{300\rho_p} \quad \text{(Exp 12.3.1B-3 US)}$$

Where, f_{ps} shall neither be taken greater than f_{py}, nor greater than (f_{se} + 30,000)
In SI system of units the associated relationship is:

$$f_{ps} = f_{se} + 70 + \frac{f'_c}{300\rho_p} \quad \text{(Exp 12.3.1B-4 SI)}$$

Where, f_{ps} shall not be taken greater than f_{py}, nor greater than (f_{se} + 210)

12.3.2 EC2 Simplified Bending Design

In addition to limitations on x that are repeated below, the following approximations are suggested. Refer to Figure 12.3.2-1

For parabola- rectangle stress-strain diagram
- For $f_{ck} \le$ 50MPa (7252 psi), $x = 0.43h$
- For $f_{ck} >$ 50MPa (7252 psi),
  $x = (1 - \varepsilon_{c2} / \varepsilon_{cu2})h$

For bi-linear stress diagram
- For $f_{ck} \le$ 50MPa (7252 psi), $x = 0.5h$
- For $f_{ck} >$ 50MPa (7252 psi),
  $x = (1 - \varepsilon_{c3} / \varepsilon_{cu3})h$

The strain parameters ε are as defined in Section 3.1.3 of EC2. For 28-day concrete cylinder strength between 12 and 50 MPa the following values may be used

$\varepsilon_{c2} = 0.0020$
$\varepsilon_{cu2} = 0.0035$
$\varepsilon_{c3} = 0.00175$
$\varepsilon_{cu3} = 0.0035$

$\lambda = 0.8$, for $f_{ck} \le$ 50MPa (Exp 12.3.2-1)

$\lambda = 0.8 - (f_{ck} - 50)/400$ for 50 < f_{ck} 90 MPa (Exp 12.3.2-2)

and

$\eta = 1{,}0$ for $f_{ck} \le$ 50 MPa (Exp 12.3.2-3)

$\eta = 1.0 - (f_{ck} - 50)/200$ for 50 < $f_{ck} \le$ 90 MPa (Exp 12.3.2-4)

f_{cd} = design value of concrete; f_{ck}/Υ_c ; and
Υ_c = concrete material factor.

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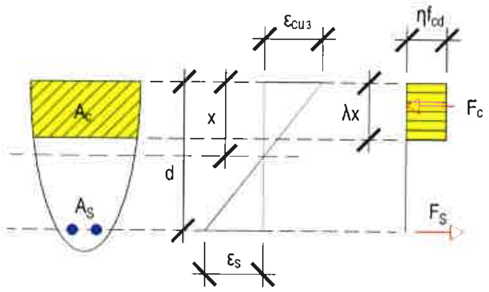

FIGURE 12.3.2 -1 Rectangular Stress Distribution for EC2 Compression Block

For bonded prestressing steel, strain compatibility is recommended to determine the stress at ultimate limit state.

For unbonded tendons [5] also the strain compatibility is recommended to compute a hypothetical strain at the level of prestressing. The hypothetical strain is adjusted using the tendon span under consideration and the total length of a tendon. In the absence of detailed computation increase in stress above the stress in tendon at service (f_{se}) by the following [6]:

- Increase service stress by 100 MPa (14.50 ksi),

A numerical example for EC2 is given in Chapter 6.

12.4 REFERENCES

ACI 318-14, (2014), *"Building Code Requirements for Structural Concrete (ACI 318-11) and Commentary,"* American Concrete Institute, Farmington Hill, MI 48331, www.concrete.org.

ACI 318-11, (2011), *"Building Code Requirements for Structural Concrete (ACI 318-11) and Commentary,"* American Concrete Institute, Farmington Hill, MI 48331, www.concrete.org, 503 pp.

European Code EC2, (2004),*"Eurocode 2: Design of Concrete Structures"* – EN 1992-1-1:2004.

[5] EC2(EN 1992-1-1:2004(E)), Sections 6.1; 5.10.5,6
[6] EC2(EN 1992-1-1:2004(E)), Sections 6.1; 5.10.8

NOTATIONS

Symbols that appear once, or occasionally, are defined at the location where they appear. The following lists the symbols that are used more frequently, but are not necessarily defined at each instance of use.

A_c	Gross cross-sectional area of concrete
A_{ps}	Cross-sectional area of prestressing steel
$A_{s,prov}$	Provided area of reinforcement in member
$A_{s,requ}$	Required amount of reinforcement
A_s	Cross-sectional area of non-prestressed steel
A_v	Cross-sectional area of shear reinforcement
c	Depth of compression zone of member in bending
d	Effective depth of member
d_t	Distance of compression fiber to farthest tension reinforcement for members in bending
E	Modulus of elasticity
E_c	Modulus of elasticity of concrete
E_{ci}	Modulus of elasticity of concrete on day; typically other than 28
E_s	Modulus of elasticity of non-prestressed steel
f'_c	Specified 28 day strength of concrete cylinder [ACI]
f_{ck}	Specified 28 day strength of concrete cylinder [EC2]
f_{cr}	Cracking stress in concrete [EC2]; same as f_{cr} and f_{ctm} in the context of this book
$f_{ct,eff}$	Tensile strength of concrete at cracking in bending [EC2][1]
f_{ctm}	mean axial tensile strength of concrete [EC2], Table 3.1[2]
f_{pk}	Yield stress of prestressed reinforcement
f_{ps}	Stress in prestressing steel at flexural ultimate strength
f_{pu}	Specified tensile strength of prestressing steel
f_r	Modulus of rupture stress; cracking stress of member in bending
f_{se}	Effective stress in prestressing steel after all losses
f_y	Yield stress of non-prestressed reinforcement [ACI]
f_{yk}	Yield stress of non-prestressed reinforcement [EC2]
f_{ywd}	Yield stress of shear reinforcement [EC2]
$f_{ywd,ef}$	Allowable stress of shear reinforcement [EC2]
h	Member depth
HYP	Hyperstatic actions, moment, shear, axial from prestressing forces (secondary actions)
K	Wobble coefficient of prestressing strand [rad/ft; rad/m]
M_{cr}	Cracking moment of a member under bending and axial force
M_n	Nominal flexural capacity of a member under axial and bending moment [ACI]
M_u	Design moment; factored moment for strength limit state [ACI]
n	Number of bars across the assumed critical section in shear design
s_l	spacing of shear bars in direction of spine;
s_t	spacing of shear bars transverse to radial direction;
U	Intensity of load from either a serviceability or strength load combination [ACI]
V_{Ed}	Design shear; factored shear force for strength limit state [EC2]
V_u	Design shear; factored shear force for strength limit state (ACI)
w_b	Intensity of lateral force exerted by a prestressing tendons; balanced load intensity
w_k	Design crack width [EC2]
x	Depth of compression zone [EC2]
β_1	Factor to determine the depth of rectangular compression zone for members in bending [ACI]
γ	Factors for reduction of material properties [EC2;TR43]
ε_c	Strain of farthest compression fiber for mem-

[1] EN 1992-1-1:2004(E), Section 7.1(2) and 7.3.2(4)
[2] EN 1992-1-12004(E), Table 3.1

ber in bending

ε_s Strain in farthest tension reinforcement for member in bending

ε_{c2} Maximum concrete strain in bending from Table 3.1 of EC2 [EC2]

ε_{cu2} Maximum concrete strain in bending from Table 3.1 of EC2 [EC2]

ε_{c3} Maximum concrete strain in bending from Table 3.1 of EC2 [EC2]

ε_{cu3} Maximum concrete strain in bending from Table 3.1 of EC2 [EC2]

μ Friction coefficient of prestressing strand between tendon and its sheathing (duct) [/rad]

ρ Reinforcement ratio

σ_c Average precompression [EC2]

φ Strength reduction factor [ACI]

ψ Factor of design load to be considered in load combination [EC2]

ω Tension reinforcement index [ACI]

ω' Compression reinforcement index [ACI]

DATA CONVERSION TABLES

TABLE D-1 ASTM Standard Reinforcing Bars (T179)

Bar size Designation*		Nominal dimensions					
		Diameter		Area		Weight or mass	
US customary	SI	in.	mm	in²	mm²	lb/ft	kg/m
#3	#10	0.375	9.5	0.11	71	0.376	0.560
#4	#13	0.500	12.7	0.20	129	0.668	0.994
#5	#16	0.625	15.9	0.31	199	1.043	1.552
#6	#19	0.750	19.1	0.44	284	1.502	2.235
#7	#22	0.875	22.2	0.60	387	2.044	3.042
#8	#25	1.000	25.4	0.79	510	2.570	3.973
#9	#29	1.128	28.7	1.00	645	3.400	5.060
#10	#32	1.270	32.3	1.27	819	4.303	6.404
#11	#36	1.410	35.8	1.56	1006	5.313	7.907
#14	#43	1.693	43.0	2.25	1452	7.650	11.380
#18	#57	2.257	57.3	4.00	2581	13.600	20.240

* Many mills will mark and supply bars only with metric (SI) designation, which is a soft conversion. Soft conversion means that the metric (SI) bars have exactly the same dimensions as the equivalent U.S. customary designation

TABLE D-2 Properties and Resistance of Prestressing Strands (T178)

Seven-Wire Strand, f_{pu} = 1860 MPa						
Size Designation		9	11	13	13 Special	15
Nominal diameter	mm	9.53	11.13	12.70	13.08	15.24
Nominal linear mass	kg/m	0.432	0.582	0.775	0.788	1.109
Nominal area	mm²	54.8	74.2	98.7	107.7	140.0
$0.7 f_{pu} A_{ps}$	kN	71.3	96.6	128.5	140.5	182.3
$0.8 f_{pu} A_{ps}$	kN	81.5	110.4	146.9	160.6	208.3
$f_{pu} A_{ps}$	kN	101.9	138.0	183.6	200.3	260.4

TABLE D-3 Properties and Resistance of Prestressing Strands (T177)

Seven-Wire Strand, f_{pu} = 270 ksi						
Nominal diameter	in.	3/8	7/16	½	9/16	0.600
Linear mass	plf	0.29	0.40	0.52	0.65	0.74
Area	in²	0.085	0.115	0.153	0.192	0.217
$0.7 f_{pu} A_{ps}$	kips	16.1	21.7	28.9	36.3	41.0
$0.8 f_{pu} A_{ps}$	kips	18.4	24.8	33.0	41.4	46.9
$f_{pu} A_{ps}$	kips	23.0	31.0	41.3	51.8	58.6

TABLE D-4 Conversion Table for US Customary System of Units to SI and MKS System of Units (T180)

American Units	If you have...	SI Units Multiply by...	SI Units To obtain...	MKS Units Multiply by...	MKS Units To obtain...
Length	in.	25.400	mm	2.5400	cm
		0.02540	m	0.02540	m
	ft	304.8	mm	30.48	cm
		0.3048	m	0.3048	m
Area	in.^{2}	645.16	mm^2	6.4516	cm^2
		6.4516E-4	m^2		
	ft^2	92903	mm^2	929.03	cm^2
		0.092903	m^2		
Force	lb	4.4482	N	0.45359	kg*
		4.4481E-3	kN	4.5359E-4	T
	kip	4448.2	N	453.59	kg
		4.4482	kN	0.45359	T
Force/Length	lb/ft	14.594	N/m	1.4882E-3	T/m
	k/ft	14.594	kN/m	1.4882	T/m
Moment	lb-ft	1.3558	N-m	13.825	kg-cm
		1.3558E-3	kN-m	1.3825E-4	T-m
	k-ft	1355.8	N-m	13825	kg-cm
		1.3558	kN-m	.13825	T-m
Stress	psi	6.8948E-3	MPa	0.070307	kg/cm^2
		6.8948	kN/m^2	0.70307	T/m^2
				7.0307E-5	T/cm^2
	ksi	6.8948	MPa	70.307	kg/cm^2
		6894.8	kN/m^2	703.07	T/m^2
				0.070307	T/cm^2
Unit weight	pcf	16.019	kg/m^3 *	16.019	kg/m^3 *
		0.15709	kN/m^3	1.6019E-2	T/m^3
Area Load	psf	47.880	N/m^2	4.8824	kg/m^2
		0.047880	kN/m^2	4.8824E-3	T/m^2
	ksf	47880	N/m^2	4882.4	kg/m^2
		47.880	kN/m^2	4.8824	T/m^2
$\sqrt{f'_c}$	$\sqrt{f'_c}$ (psi)	0.083035	$\sqrt{f'_c}$ (MPa)	0.26515	$\sqrt{f'_c}$ (kg/cm^2)

TABLE D-5 Conversion Table for SI System of Units to American Customary System of Units (T181)

SI Units	If you have...	American Units Multiply by...	American Units To obtain...	MKS Units Multiply by...	MKS Units To obtain...
Length	mm	0.039370	in	0.1	cm
		3.2808E-3	ft	0.001	m
	m	39.370	in	100	cm
		3.2808	ft	1.0	m
Area	mm^2	1.5500E-3	in^2	0.01	cm^2
		1.0764E-5	ft^2		
	m^2	1550.0	in^2	10000	cm^2
		10.764	ft^2		
Force	N	0.22481	lbs	0.10197	kg *
		2.2481E-4	kips	1.10197E-4	T
	kN	224.81	lbs	101.97	kg
		0.22481	kips	0.10197	T
Force/Length	kN/m	68.522	lb/ft	0.10197	T/m
		0.068522	k/ft		
Moment	N-m	0.73756	lb-ft	10.197	kg-cm
		7.3756E-4	k-ft	1.0197E-4	T-m
	kN-m	737.56	lb-ft	10197	kg-cm
		0.73756	k-ft	0.10197	T-m
Stress	MPa	145.04	psi	10.197	kg/cm^2
		0.14504	ksi	101.97	T/m^2
				0.010197	T/cm^2
	kN/m^2	0.14504	psi	0.010197	kg/cm^2
		1.4504E-4	ksi	0.10197	T/m^2
				1.0197E-5	T/cm^2
Unit weight	kg/m^3	0.062427	pcf	0.001	T/m^3
	kN/m^3	6.3659	pcf	101.97	kg/m^3
Area Load	N/m^2	0.020885	psf	0.10197	kg/m^2
		2.0885E-5	ksf	1.0197E-4	T/m^2
	kN/m^2	20.885	psf	101.97	kg/m^2
		0.020885	ksf	0.10197	T/m^2
$\sqrt{f'_c}$	$\sqrt{f'_c}$ (MPa)	12.043	$\sqrt{f'_c}$ (psi)	3.1933	$\sqrt{f'_c}$ (kg/cm^2)

TABLE D-5 Conversion Table for SI System of Units to American Customary System of Units (T181)

MKS Units		**American Standard** Units		**SI** Units	
	If you have...	**Multiply by...**	**To obtain...**	**Multiply by...**	**To obtain...**
Length	cm	0.39370	in	10	mm
		0.032808	ft	0.01	m
	m	39.370	in	1000	mm
		3.2808	ft	1.0	m
Area	cm²	0.15500	in²	100	mm²
		1.0764E-3	ft²		
	m²	1550.0	in²	1.0E+6	mm²
		10.764	ft²		
Force	kg	2.2046	lbs	9.8067	N
		2.2046E-3	kips	9.8067E-3	kN
	T	2204.6	lbs	9806.7	N
		2.2046	kips	9.8067	kN
Force/Length	kg/m	0.67197	lb/ft	9.8067	N/m
		6.7197E-4	k/ft	9.8067E-3	kN/m
	T/m	671.97	lb/ft	9806.7	N/m
		0.67197	k/ft	9.8067	kN/m
Moment	kg-cm	0.072330	lb-ft	0.098067	N-m
		7.2330E-5	k-ft	9.8067E-5	kN-m
	T-m	7233.0	lb-ft	9806.7	N-m
		7.2330	k-ft	9.8067	kN-m
Stress	kg/cm²	14.223	psi	0.098067	MPa
		0.014223	ksi	98.067	kN/m²
	T/cm²	14223	psi	98.067	MPa
		14.223	ksi	98067	kN/m²
Unit weight	kg/m³	0.062427	pcf	9.8067	kN/m³
	T/m³	62.427	pcf	1000	kg/m³
Area Load	T/m²	204.82	psf	9806.7	N/m²
		0.20482	ksf	9.8067	kN/m²
$\sqrt{f'_c}$	$\sqrt{f'_c}$ (kg/cm²)	3.7714	$\sqrt{f'_c}$ (psi)	0.31316	$\sqrt{f'_c}$ (MPa)

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POST-TENSIONED BUILDINGS

ABOUT AUTHOR

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A practicing structural engineer and software developer, Dr. Bijan Aalami, Professor Emeritus of Civil Engineering at San Francisco State University, is an internationally renowned educationalist and leader in the analysis and design of concrete structures with specialization in post-tensioning. A native of Iran, he graduated from University of London (Imperial College), held positions as professor and Vice-Chancellor of Arya-Mehr (now Sharif) University, Chancellor of former Abu-Reyhan University, followed by appointment as professor of Civil Engineering at San Francisco State University.

Professor Bijan has made decisive contributions to the understanding, behavior and design of post-tensioned buildings and bridges. Bijan was recognized as the first Life Member of the Post-Tensioning Institute, for his decades of contribution to the post-tensioning industry. A member of the former Imperial Academy of Sciences in Iran, a Life Member of ASCE; honorary member of the Argentine Institute of Structural Engineers, he has been holding educational courses on design of concrete buildings and bridges throughout the United States, Canada and over 30 other countries worldwide. Bijan is a Chartered Engineer in the UK, and a registered Structural Engineer in California. He received the American Concrete Institute's Design Award (2003) for the "application of advanced engineering to a notable concrete structure." In 2003, Bijan's contribution to the industry was recognized through his initiation in the Hall of Fame of the Post-Tensioning Institute.

As a Principal of ADAPT Corporation – a structural engineering firm in California that specializes in design of concrete structures, and provides design software for consulting firms, Bijan has been actively engaged for over 30 years in the design and construction of numerous concrete buildings, bridges and special structures, in particular post-tensioning. A world teacher, Bijan has enriched the practice of engineers in North and Latin America, Far East, Europe and the Middle East. His extensive publications on concrete design, in particular post-tensioning, are a treasured resource for practical design of conventionally reinforced or post-tensioned buildings and bridges. Dr Bijan is the founder, and for over 20 years was the project leader of the software suite ADAPT that is serving the concrete design engineers in over 75 countries worldwide.

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